Volu

Chapter

Astro-
spectroscopy
SHAIN

Stud.Obsy.

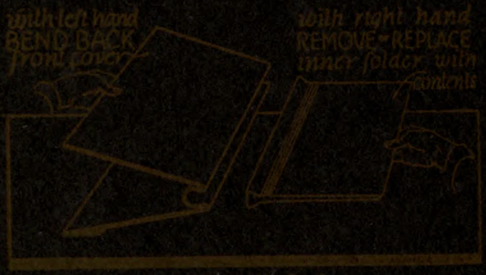
QB461 Gerasimovich
K872 Course in astrophysics and
a
copy [illegible] stellar astronomy. Translation.
1:2

YE17842

THE ELBE LINE

No. 27

SPRING BINDER



MADE IN U. S. A.

THE UNIVERSITY OF CALIFORNIA LIBRARY

Please sign name and
date and file card in Bx
s.e. corner of Room ~~11~~. 21.
When you return binder,
cross off your name and
put card in binder again.
nm

QB461
K8T2
v1:2
cop a
Astron
Library

Chapter II

Spectroscopy

by [illegible]

1. The Spectrograph. General Theory. The application of the spectrum analysis
astronomy began about the year 1860. At first a prism placed in the objec-
e was used, and the spectrum was observed in the focal plane by means of an
epiece. However, it soon became apparent that for the determination of wave-
ngths of spectral lines it is necessary to have for purposes of comparison
spectrum of a terrestrial element beside the spectrum of a celestial body.
this connection, and also because of the requirements of greater purity of
spectrum, spectroscopes having collimators and slits were introduced. Be-
ides, a spectroscope of direct vision was also used. In order to obtain a
eater resolving power in the investigation of the solar spectrum a spectro-
ope with a diffraction grating is used. The introduction of photography in
tronomy has opened a new era in the application of the spectroscope. A
ectroscope with a photographic camera is called a spectrograph.

The spectrograph, a powerful instrument for astrophysics, is used only in
bination with the telescope which, in this case, should be regarded only as
n additional part of the spectrograph. In describing the spectrograph we
al not go into geometrical and physical optics as given in physics, but shall
ly stress a few ideas and details which are of great importance from the
ronomical point of view.

The spectrograph is characterized by the following four quantities: 1)
ngular dispersion, 2) linear dispersion, 3) resolving power, and 4) purity
spectrum.

1) Dispersion and Resolving Power. Angular dispersion is the quantity $\frac{d\theta}{d\lambda}$
ch characterizes the change in the direction of rays in their passage through
rism within the limits of a narrow section of wave-lengths (θ is the angle
deviation). The angular dispersion depends only on the properties of the
sm. Under minimum deviation

$$\frac{d\theta}{d\lambda} = \frac{dn}{d\lambda}\frac{2\sin\frac{A}{2}}{\sqrt{1-n^2\sin^2\frac{A}{2}}}$$

re A is the angle of refraction, n is the index of refraction.

Linear dispersion is the quantity $\frac{dl}{d\lambda} = F\frac{d\theta}{d\lambda}$ where F is the focal
tance of the objective of the camera expressed in millimeters, and $\frac{d\theta}{d\lambda}$ is
xpressed in seconds of arc. The linear dispersion shows the number of Å
in 1mm of the length of the spectrum near the given wave-length.

The resolving power of a prism $\frac{\lambda}{\Delta\lambda}$ characterizes that difference in wave-
gth $\Delta\lambda$ for which two neighboring lines may be separated when the slit of
spectrograph is infinitely narrow. This quantity depends only on the dimen-
sions of the prism. It is easy to show that the resolving power of a prism

$\frac{d\theta}{d\lambda}$ $\frac{1}{206000}$

1. Spectroscopy

unacc.

QB465

Chapter II

Astrospectroscopy

By G. A. ~~Shine~~ Shain

1. The Spectrograph General Theory. The application of the spectro scope to astronomy began about the year 1860. At first a prism placed on the objective was used, and the spectrum was observed in the focal plane by means of an eye-glass. However, it soon became apparent that for the determination of wave-lengths of spectral lines it is necessary to have for purposes of comparison the spectrum of a terrestrial element beside the spectrum of a celestial body. In this connection, and also because of the requirements of greater purity of the spectrum, spectroscopes having collimators and slits were indroduced. Besides, a spectroscope of direct vision was also used. In order to obtain a greater resolving power in the investigation of the solar spectrum a spectroscope with a diffraction grating is used. The introduction of photography in astronomy has opened a new era in the application of the spectroscope. A spectroscope with a photographic camera is called a spectrograph.

The spectrograph, a powerful instrument for astrophysics, is used only in combination with the telescope which, in this case, should be regarded only as an additional part of the spectrograph. In describing the spectrograph we shall not go into geometrical and physical optics as given in Physics, but shall merely stress a few ideas and details which are of great importance from the astronomical point of view.

The spectrograph is characterized by the following four quantities: $\frac{d\theta}{d\lambda}$ 1) angular dispersion, 2) linear dispersion, 3) resolving power, and 4) purity of spectrum.

1) Dispersion and Resolving Power. Angular dispersion is the quantity $d\theta$ which characterizes the change in the direction of rays in their passing through a prism within the limits of a narrow section of wave-lengths $d\lambda$ (θ is the angle of deviation). The angular dispersion depends only on the properties of the prism. Under minium deviation

$$\frac{d\theta}{d\lambda} = \frac{d\mu}{d\lambda} \cdot \frac{2\sin\frac{A}{2}}{\sqrt{1-\mu^2\sin^2\frac{A}{2}}}, \qquad (1)$$

where A is the angle of refraction, μ is the index of refraction.

Linear dispersion is the quantity $\frac{d\theta}{d\lambda}\cdot\frac{f}{206000}$ where f is the focal distance of the objective of the camera expressed in millimeters, and $\frac{d\theta}{d\lambda}$ is expressed in seconds of arc. The linear dispersion shows how many angstroms are in 1mm of the length of the spectrum near the given wave-length λ.

The resolving power of a prism $\frac{\lambda}{\Delta\lambda}$ characterizes that difference in wave-length for which two neighboring lines may be separated when the slit of the spectrograph is infinitely narrow. This quantity depends only on the properties of the prism. It is easy to show that the resolving power of a prism is

$$\frac{d\theta}{d\lambda} : \frac{f}{206000}, \qquad (2)$$

where t_1 and t_2 are the longest and shortest paths of the rays in the prism and μ is the index of refraction. Since t_2 is usually equal to zero and t_1 is close to the length of the base of the prism, it is evident that in order to increase the resolving power it is desirable to use larger prisms. It also follows that the resolving power is proportional to the number of prisms in a spectrograph . The resolving power is expressed in units which are 1000 times as large as the usual ones; for instance instead of 30000 we write 30. Of some interest is the following formula

$$\frac{d\theta}{d\lambda} = \frac{R}{a}, \tag{3}$$

which shows the relation between the angular dispersion, the resolving power and the size of the prism (α is the diameter of a pencil of rays falling on the prism).

The conception of resolving power is related to infinitely thin lines and a infinitely narrow slit. As such things never obtain in practice, the conception of the purity of spectrum is introduced which characterizes that difference of wavelengths of two neighboring (monochromatic) lines at which the latter still remain separated for a given finite width d of the slit. Schuster gives the following simple formula which expresses the relation between the purity P, resolving power R, width of slit d, and angle φ under which the collimator of the objective is seen from the center of the slit:

$$P = \frac{\lambda}{\varphi d + \lambda R} \tag{4}$$

The derivation of this formula is based on Rayleight's theorem according to which two close lines are separated when the maximum of intensity of the diffractional image (each monochromatic line represents a diffractional image of the slit) of one of them falls upon the first minimum of the second line. The distance of the first minimum l from the chief maximum is determined from the formula

$$\frac{a_1 l}{\lambda f} = 1, \tag{5}$$

where a_1 is approximately the width of the pencil of rays entering the objective of the tube or camera, and f is the focal distance of the latter.

Indeed the resolving power and the purity of the spectrum are determined by the superposition of diffractional phenomena occuring because of the passing of the pencil of light through the slit and of the limitation of the aperture of the optical parts of the spectrograph. In the case of an infinitely narrow slit the distribution of intensity in the diffractional image is expressed by the formula known from Physics:

$$i(\xi) = \frac{\sin^2 \xi}{\xi^2} \tag{6}$$

If the slit has a finite width d we shall have:

$$I(\chi) = \int_{\chi-\frac{\sigma}{2}}^{\chi+\frac{\sigma}{2}} \frac{\sin^2\alpha}{\alpha^2}\, d\alpha, \qquad (7)$$

where $\chi = \frac{Bl}{f\lambda}\pi$; $\sigma = \frac{Bd}{f\lambda}\pi$; l -- is the distance, expressed in linear measure, in the plane from the center of the diffractional image; d is the width, expressed in linear measure, of the slit; f and f' are the focal distances of the camera and of the collimator; B is the width of the pencil determined by the size of the prism (in the computation of x and σ B may be taken, with sufficient approximation, as equal to the diameter of the camera and collimator). The unit of distance is so selected that the first minimum is at $\chi = \pi$

It is possible to compute by tables based on mechanical integration (Schuster, Ornstein, Minnaert), the distribution of intensity in the diffractional image of the slit for a given spectroscope. The computation of this "instrumental contour" (digressing from the imperfections of the optical parts of the spectrograph) is of great interest from the spectrophotometric point of view since the observed contour of the spectral lines is a result of imposition of the real and instrumental contours.

It is to be noted that when we deal with a photographic plate the conception of resolving power and of the purity of spectrum becomes somewhat more complicated because of the grains of a plate have a definite size, and the distance between them may change under different conditions. The photographic effect cannot be accurately evaluated. In this respect the matter is much better with long cameras.

2. The Loss of Light in the Spectrograph. In spectral investigations we have such great losses in brightness, which may reach 98 °/o in a spectrograph of three prisms, that, in this case, only 1/50 part of incident light from a star is used. Even before reaching the slit the light of a star is weakened by a great percentage (sometimes more than one-half) because of absorption and reflection in the objective of a refrator or because of reflection in the case of a reflector. Next, because of spherical or zonal aberration of diffraction as well as of scattering and other causes, a star on the slit appears in the form of noticeable disk even under ideal atmospheric conditions.

Objectives or mirrors of a large size which are usually used in combination with the spectrograph may be considered good when the diameter of the stellar disk in focus does not exceed 0".8 - 1".0, which at a focal distance, let us say, of 16m corresponds approximately to 0.07mm. In stellar spectrography we usually work with a slit having a width of 0.035 to 0.050mm. It appears, therefore, that even under ideal atmospheric conditions we have a loss at the slit in the sense that the entire light cannot be utilized by the spectrograph. This happens because of the limited size of the objective or mirror and the imperfection of the way they are made. If we add that on

account of atmospheric variations this ideal disk of 0".8 sometimes widens into 5" or even more, it becomes apparent that the loss at the slit alone sometimes reaches 80 °/o and more.

We shall speak here in greater detail of those losses which take place in the spectrograph itself. Much light is lost because of absorption and reflection in separate optical parts of the latter.

As to absorption, it is subject to Lambert's law

$$I' = Ia^d,$$

where I and I' are the intensity before entering some medium and after coming out from it, d is the thickness of the layer, and a is the absorption per unit of thickness.

Hartmann recommends decreasing somewhat the altitude of the prism or shifting it so that the base should remain a little unused and the vertex should remain somewhat lower than the edge of the passing pencil. In this, way the losses from absorption decrease.

Having in mind to obtain as little lose of light as possible, the more transparent sorts of glass have been used, although light is lost in dispersion. Table 3 shows the transmiting power of prisms made of various kinds of glass reduced to the same conditions with respect to dispersion.

This table illustrates the great losses in light especially when the spectrograph is of three prisms. The absorption depends on the wave-length and is usually rises rapidly for shorter waves. We must also take into consideration the absorption increases considerably toward the base of the prism which fact makes the outgoing monochromatic pencils of rays somewhat asymmetric with respect to the distribution of light; and this may under certain circumstances cause a systematic error in the determination of radial velocities of stars. The absorption in the objectives of the collimator and of the camera is very small as compared with the absorption in prisms, especially when the spectrograph is of many prisms. In the passing of light through the spectrograph considerable losses take place due to the reflection from the surfaces of the prisms and lenses. In order to take into consideration this loss it is necessary to keep in mind that a refracted ray of light is polarized at each reflection, and therefore it behaves henceforth as an "ordinary" ray.

It is not difficult to show that in passing through m prisms set in a position of least deviation the amount of light passed will be equal to

$$\frac{1}{2}\left[(1-A)^{2m} + (1+B)^{2m}\right]$$

where A and B are the reflected parts of light (at each refraction) which is polarized in the plane of falling and which is perpendicular to it. These parts

Table 3

inds of Glass (#of Jena Glass)	Number of Prisms	Transmission Ability
. V. 3248, Ultra - violet flint	1	0.822
.722, light barytes flint	1	0.657
.578, barytes (heavy spar) flint	1	0.636
.118, Usual flint	1	0.756
.118, Usual flint	2	0.603
.118, Usual flint	3	0.503
.102, Heavy flint	1	0.467
.102, Heavy flint	2	0.235
.102, Heavy flint	3	0.126

of A and B are computed by Frenel's formulae. The most favorable case in this respect obtains when the prisms are placed in the position of minimum deviation which is equal to the angle of polarization. Table 4 shows the losses for prisms of angle of refraction of 60° and of an index of refraction of 1.6 when these prisms are placed in the position of the minimum deviation which is equal to the angle of polarization (about 64°). The loss in the prisms due to reflection does not exceed 50°/o even for a large number of prisms.

Table 4

Number of prism	Losses due to reflection
1	0.147
2	0.252
3	0.328
10	0.509

The loss of light in lenses of the optical system is also considerable. Since the rays fall here perpendicularly, Frenel's formula for the loss of light gives for a surface for $n = 1.55$ a loss $q = 0.046$. Taking into consideration that the objectives of the collimator and camera consist of two and more lenses we shall find that the loss through reflection may reach a large percentage. In order to decrease these losses the lenses of the collimator or the objective are glued together or the space between them is filled by some transparent medium (the index of refraction of which is equal to that of the glass) thus reducing the number of reflecting surfaces. Summing up the action of all effects causing the loss of light we must arrive at the melancholy conclusion that only a small fraction of stellar light collected by the objective or mirror falls directly upon the plate intercepting the spectrum, and this small fraction diminishes still more as the number of prisms in the spectrograph increases. Because of these large dispersions in prisms the diffraction grating may successfully compete with the prism as to their respective usfulness.

3. The Brightness of a Spectrum. Let us now investigate the instrumental factors, determining the brightness of the spectrum. Let the slit of width d and of length h be uniformly illuminated by a source of light of brightness I. Let us denote the corresponding quantities in the focal plane of the tube or camera by d', h', and i. Then

$$i = I\varepsilon \frac{hd}{h'd'}$$

Factor ε is the loss due to absorption and reflection. If we denote f' and f the focal distances of the collimator and camera, it is evident that

$$d' = \frac{f}{f'} d + H$$

and

$$h' = \frac{f}{f'} h. \qquad (9)$$

When the width of the slit is more than 0.02mm. we may neglect the widening connected with the phenomenon of diffraction. In formula (9) H denotes the widening caused by the dispersion whenever we deal with non-monochromatic light. Substituting (9) and (8) we obtain

$$i = I\varepsilon\left(\frac{f'}{f}\right)^2 \frac{1}{1+\frac{fH}{fd}} = I\varepsilon\left(\frac{\beta}{\psi}\right)^2 \frac{1}{1+\frac{\beta H}{d\psi}}, \qquad (10)$$

where $\psi = \frac{a}{f'}$ and $\beta \frac{a'}{f}$ (a and a' are the diameters of the objectives of the collimator and the camera). Let us consider two cases: 1) monochromatic radiation and 2) non-monochromatic radiation. In the first case H = 0 or very close to zero, and, therefore, practically:

$$i = I\varepsilon\left(\frac{\beta}{\psi}\right)^2 \qquad (11)$$

i.e. for a spectrum consisting of monochromatic radiation the brightness of lines does not depend on the width of the slit, if the slit is not very narrow. This result must be understood as follows: widening the slit we increase the quantity of light passing through the spectrograph, but the width of the line also is increased proportionately thereby; therefore the surface brightness of the lines remains unchanged. For a constant width of the slit the surface brightness is proportional to the square of the ratios of the focal distances of the collimator and of the camera.

In the band spectrum or in the continous spectrum H is not large. If the source radiates a region from λ_1 to λ_2 it is evident

$$H = f\frac{d\theta}{d\lambda}(\lambda_1 - \lambda_2) = \frac{R}{\beta}(\lambda_1 - \lambda_2). \qquad (12)$$

The latter follows from formula (3). Therefore:

$$i = I\varepsilon\left(\frac{\beta}{\psi}\right)^2 \cdot \frac{\psi d}{R(\lambda_1 - \lambda_2)}.$$

Equation (13) shows that the brightness of the continuous or band spectra is proportional to the width of the slit and is inversely proportional to the

resolving power.

The computation of the brightness of the spectrum as a function of the constants of the spectrograph is somewhat difficult. Two cases need be considered here:

1) when the object under investigation is a star, and

2) when the object is elongated, such as a nebula or a comet, for instance.

During the spectrophotographing of stars the slit is not uniformly illuminated, and the result will depend on the quality of the image. When the slit is opened a proportional increase of brightness within certain limits may be expected. The brightness of the spectrum depends chiefly on the quality of the image, i.e. on the surface brightness of the image (tremor disc). With very good images when the diameter of the disk is of the order of 1", it is necessary to "guide" the star along the slit for the widening of the spectrum; and, when the disk is of the order of 3" to 4" and more, the spectrum need not be widened. All these considerations make the calculation of the exposure time difficult, and therefore an empirical method is used. For the continuous spectrum the widening of the slit up to certain limits gives a proportional increase of brightness if we use a slit which is neither too narrow nor too wide; in the latter case the width of the slit is determined by the optical qualities of the mirror or objective as well as by the atmospheric conditions. For a spectrum consisting of separate monochromatic radiations the widening of the slit (if we neglect the phenomena of diffraction) does not increase the surface brightness of the lines. In both cases the brightness of the spectrum of stars increases as the focal distance of the camera decreases, however the rate of increase depends on the quality of the image. Because of a smaller scale with short cameras it is sometimes necessary to "guide" the star on the slit linearly by a larger amount.

In the case of elongated objects the formulae (11) and (13) remain approximately correct.

4. The Spectrograph with the Diffraction Grating. The diffraction grating spectrograph is chiefly used for the study of the Sun. However, even in stellar spectroscopy this type of spectrograph is becoming ~~of~~ important. Without entering into the theory of the diffraction grating, which may be found in a course on optics, we shall merely speak here briefly about those characteristics which are of astronomical interest: dispresion, resolving power, and brightness of spectrum. These conceptions are the same as those for the prism. The resolving power in the case of the grating is

$$R = \frac{\lambda}{\Delta\lambda} = N m,$$

when N is the number of lines (on the grating) and m is the order of the spectrum.

In a Rowland grating there are from 10000 to 20000 evenly spaced lines per inch, and althogether there are sometimes as many as 280000 lines on a single grating.

The grating has a great advantage as compared with the prism in regard to resolving power. Taking m sufficiently large we may obtain a very large

value for R. When a large resolving power is required gratings instead of prisms are almost alway used in spite of disadvantages in other respects that the gratings may offer. In regard to dispersion the diffraction grating is superior to the prism for the same reason. Further, in case of ultraviolet and infrared rays for which the ordinary prisms are not sufficiently transparent, or even opaque, the advantage remains with the grating, especially when a concave grating is used which requires no additional optical glass.

In speaking about dispersion here, we have in mind the "normal" dispersion which under certain conditions is realized in a spectrograph having a grating. In this case the property that to equal linear distances in the spectrum correspond equal sections of wave-lengths is very valuable. The application of the grating spectrograph to the study of red and infrared parts of the spectrum is very advantageous as compared with a prism, since in the latter case the dispersion rapidly diminishes as the wave-length increases.

As to the brightness of the spectrum produced by the grating, it is natural to expect great losses because of distribution of light in spectra of a various orders. The problem of distribution of brightness in the spectra cannot be accurately solved theoretically, for in reflecting gratings, used in astrophysics, of great importance are not only the width but also the form of the cross-section of these little furrows by which the bands of the reflecting surface are differentiated one from another. The slight application of spectrographs with gratings in stellar spectroscopy may be explained by these losses.

However, the introduction of new methods of ruling the fine lines, in which a special shape is given to the cross-sections of these lines, results in the concentration of a large part of the light (of the order of 50 °/o) in one spectrum of the given order.

There are several kinds of grating spectrographs. In stellar spectroscopy the most suitable grating spectrograph is the one whose parts are arranged as those of a prism spectrograph. The rays from the illuminated slit coming out from the collimator in parallel lines fall on the flat grating at a small angle, and the reflected diffractional image is received by the camera placed at a certain angle on the same side as the collimator.

Of another type is the Littrow spectrograph (see ch. V). The diffraction image is sent back into the objective of the collimator after the reflection from the plane grating. The slit is situated not exactly on the axis of the collimator, but somewhat above or below it so that the diffraction image is obtained at the other side of the axis.

Finally, spectrographs with a concave grating are used which give accurately a "normal" spectrum and are quite suitable for the absolute measurement of wave-lengths. The classical work of Rowland was done by a spectrograph of this kind.

We shall interrupt our discussion of this type of spectrograph in order to show how the wave-lengths are determined.

The wave theory of light gives the following expression for the plane reflective grating:

$$(a + b)(\sin\varphi - \sin\psi) = m\lambda \qquad (15)$$

where a + b is the distance between the centers of two neighboring ruled fine lines (the constant of the grating). The angles ψ and φ correspond to the direction of incident and reflected diffraction rays, and m determines the order of the spectrum.

Formula (15), as is known, gives the difference of the speed of rays for various wave-lengths.

This formula is also applied in the case of concave grating. A concave spherical grating has the following property: if we construct a sphere, the diameter of which is the radius of curvature of the grating, and this sphere is tangent to the grating at its center, then, placing the slit parallel to the lines of the grating, we shall find that the loci of the maxima of the diffraction image will be located on that sphere. Thus the concave grating gives a series of real images and it has the properties of concave mirror and of a diffraction grating simultaneously. In accordance with this property Rowland's mounting may be represented schematically as follows: on two strong rails AB and AC, fastened to each other at a right angle (fig. 18) roll two carriages which support the rod BC. At the end B of the rod is placed the concave grating and at the other end C is placed a camera. The vertical slit ~~is slit~~ is fixed at A. The distance BC is equal to the radius of curvature ρ of the grating. As the carriages move on the rails, B and C move along the rails, so that the points A, B, and C remain always on the circle whose diameter is ρ. Various parts of the spectrum may be brought to C.

Turning to formula (15) we see that at $\varphi = 0$

$$(a + b) \sin \psi = m \lambda \tag{16}$$

Since AC = $\rho \sin \psi$

$$\lambda = \frac{(a+b)}{m \rho} AC \tag{17}$$

This is the condition for the "normal" spectrum, since the wave-length is proportional to AC. If the position of any one line of a well-known wave-length be marked on the scale AC, then all the other lines are determined on the same scale, for which purpose we need to know the coefficient of proportionality determined from observations. All this remains true for spectra of any order. The determination of wave-lengths by means of the diffraction grating is reduced first of all to a more accurate determination of a standard line. This is obtained, as the formula shows, by careful and numerous measurements of the angles of diffraction and the constant of the grating. The standard line at first was the sodium line D (5896.156), and later the cadmius line m ✓ λ 6438.4696 was selected. For a more accurate determination of the absolute wave-length of the standard line Michelson's method of the echelon interferometer is used (improved by Farbry and Perrot) which is described in textbooks on optics.

In speaking of the application of the spectrograph with a diffraction grating to photographing stellar spectra, it is necessary to compare this type with the usual prism type of spectrograph. The ultra-violet and the extreme violet as well as the red and infrared parts of the spectrum are little accessible and unsuitable regions of investigation by means of the prism spectrograph: the former because of insufficient transparency of the prisms and the objectives for short wave-lengths, and the latter because of the diminution of dispersion with the increase of wave-length and also because of insufficient transparency for the infrared rays. If for a given spectrograph the dispersion near H and K is equal to 20A per 1mm, then usually near $H\alpha$ the dispersion will be more than 100A per 1mm. For the investigation of the red and infrared parts of the spectrum for any moderate dispersion it is necessary to use a much greater dispersion for the usual part of the spectrum. This may be obtained by an increase of the number prisms, but in this case the light-gathering power of the spectrograph greatly diminishes. The lengthening of the camera is also connected with great losses in light-gathering power and also with mechanical difficulties. These inconveniences are so great that the red and infrared parts of the spectrum of stars up to the present has been left almost entirely uninvestigated with the usual spectrographs.

In this respect the spectrograph with the diffraction grating has an indisputable advantage; its slight application to the study of red and infrared portions of the spectrum is due to somewhat exaggerated fears as to the losses of brightness.

In regard to the usual photographic part of the spectrum the comparison of both types of spectrographs leads to the following conclusions:

1) For a very large dispersion of the order of 1Å per 1mm near $H\gamma$ the grating is more advantageous than the prism, since in the latter the losses of brightness increase greatly because of a large number of prisms. Such a dispersion may be applied to the Sun and to the detailed study of the spectra of the bright stars.

2) For a dispersion of the order of 10Å per 1mm near $H\gamma$ prisms are more advantageous for the photographic part of the spectrum near $\lambda = 4200 - 4800$A, but they are not as good as gratings near the lines H and K and farther toward the short wave-lengths.

3) For a dispersion of the order of 30Å per 1mm and less near $H\gamma$ a one-prism spectrographs is much more advantageous than a diffraction grating, except in the red and infrared region for reasons indicated above.

The application of the grating spectrograph to the study of stars is rendered more difficult by the influence of bending. As we shall subsequently see, the influence of bending is very great, and in this respect the grating is worse than the prism. It is known that a small shift of the prism when the latter is placed in a position of minumum deviation has a minimum effect on the passing of rays whereas for a reflecting grating the effect of the shift will be doubled. This circumstance requires careful attention from the point of view of mechanical construction; however, this obstacle may be overcome. The positive aspect is the greater compactness of the grating spectroscope.

If the bending is removed the advantageous side of the application of the grating to the determination of radial velocities is that the linear shift of lines for a given velocity will increase with the wave-length, as follows from the Doppler-Fizeau principle. For instance, for $H\alpha$ the shift will be

approximately one and a half times as large as for $H\gamma$. Thus, contrary to what we had in the case of the prism, the weights of the determined radial velocities for individual lines will increase toward the red end of the spectrum. Another advantage of the grating is that the obtained spectrum is a "normal" spectrum, and therefore that unnatural visible distribution of brightness along the spectrum is absent here, whereas it occurs in the case of a prism spectrograph because of the diminution of the dispersion toward the red end and also because of the selective absorption in prisms. The entire spectrum may be obtained on the plate without overexposure or underexposure which is very valuable for spectrophotometric investigations. A deficiency in the grating is the existence of so-called "ghosts" or false lines the source of which is to be looked for in the periodic error in the ruling of the fine lines on the grating by the ruling machine. In certain cases this deficiency is so great in producing much scattering of light that the contrast of lines greatly diminishes. This circumstance which distorts the contour of spectral lines is very dangerous in spectrophotometric investigations.

It follows from the above that the spectrograph with a diffraction grating may be applied most successfully to the study of the red and infrared parts of the spectrum of stars, to the solution of problems which are unsuitable for the prism spectrograph, for instance: 1) the investigation of radial velocities of the fainter red stars; in the photographic region their spectrum is very faint, and for the same dispersion for the same instrument the field of investigation widens two times if we use the lines in the red part of the spectrum; 2) the study of "forbidden" lines which are in the infrared part of the spectrum, 3) the study of molecula spectra in the red and infrared region; 4) the indentification of lines in this little known region of the spectrum; 5) the study of the distribution of brightness of the continuous spectrum in the red and infrared region.

On the other hand, an excellent application of the diffraction grating spectrograph is to the spectrophotometric investigations of spectral lines with a very large dispersion.

2. Details of the Spectrograph Mechanical Construction. Mounting.

Let us pass on to the description of the separate parts of the spectrograph and of the requirements to which it is subject.

5. The Slit. The Comparison Spectrum. The slit is one of the most essential parts of the spectrograph. The faces of the slit are usually prepared of polished nickel or mirror metal, and one of the faces is made immovable while the other can be shifted by means of a micrometric screw. The width of the slit may be measured by the subdivisions on the head of the screw. The edges of the slit must be sufficiently sharp; in order to prevent damage when the slit is closed a stopper is sometimes used. The upper part of the collimator together with the slit may rotate within certain limits. This enables us to make the slit parallel to the refracting edge of the prism and thus correct the inclination of the spectral lines so that they become perpendicular to the length of the spectrum. A strict perpendicularity of lines contributes to the accuracy of measurement of their positions; besides, it is a necessary condition in the photometry of spectral lines. This condition also simplifies the reduction for curvature of spectral lines. Usually there is also a possibility of correction for parallelism of the two halves of the slit (a strict parallelism of the edges of the slit is a necessary condition in spectrophotometric investigations).

Depending on the method of adjustiment the plane of the slit is made either perpendicular, or inclined at a small angle, to the main optical axis of the instrument. In the first case the rays passing through the collimator and those partially reflected from the surface of the first and second prism may be observed in the form of a star or spectrum with the aid of a telescope placed at the side. In this method of adjustment the field of view is small, and this leads to numerous inconveniences. In the new spectrographs the adjustment is usually made according to the method introduce by Huggins.

The essence of this method is easily understood by observing figure 19 Because of a certain inclination of the plane of the slit a part of the rays (of a star), which did not pass into the collimator and which are reflected from the faces S of the slit, may be observed together with the slit on the side by a telescope O_1 P_2 O_2 P_3 E as is shown in figure 19. In this case the adjustment is much more convenient.

With the slit is connected a special kind of diaphragm by means of which the objective of the collimator may be illuminated by the rays from the star or by light from an artificial source (mostly by a voltaic arc with F_e or T_i electrodes). It is very important that this diaphragm, as well as the other parts needed for adjustment or for obtaining of an artificial spectrum be separated from the slit itself. While shifting the diaphragm no pressure of any kind should be exerted either on the slit or on the upper part of the collimator, for a relative shift of the spectral lines and the artificial source may otherwise take place.

The diaphragm, the device for its shift, and the method of illumination from an artificial source are different in various spectrographs. Often a diaphragm which may slide in front of the slit is a plate with two cut-outs: one in the shape of an isosceles triangle, the other is either above or below on the same plate limiting the former. Depending on the position of diaphragm either the central of slit of this or that length or the side parts of the slit for the obtaining of artificial spectrum, symmetrically situated with respect to the stellar spectrum, are opened. Besides, there is a possibility of placing the spectrum of F_e or T_i on this or that distance from the stellar spectrum. In the contrivance for the movement of the diaphragm there are usually some plugs which enable us to regulate the length of the central part of the slit for the star as ~~the~~ well as the distance of the spectra of the artificial source of light. In the older spectrographs the artificial source of light is placed immediately in front of the slit of the collimator, and, in order to obtain a more uniform illumination of the objective of the collimator, a milky glass is placed behind the source of light during the burning of the voltaic arc producing an artificial spectrum. In the newer spectrographs

the source of the comparison spectrum is placed at the side, and by means of lenses and prisms the rays find their way into the slit of the collimator and illuminate uniformly its objective. In most cases the auxiliary optics and diaphragms are so placed that the spectrum of the arc may be photographed as many times as necessary without interrupting the exposure of the star.

The slit must be placed exactly in the focus of the telescope as well as of the collimator simultaneously.

The methods of bringing the slit into the focus of the telescope are different depending on whether a reflector in the Cassegrain form or a refractor or a reflector in the Newtonian form is used. In the first case it is preferable to shift carefully the hyperbolic mirror, without touching the spectrograph, (keeping in mind that a small shift of this mirror produces a considerable shift of the common focus) until we obtain on the slit a sharp image of the star. In the second case it is desirable to be able to shift the entire spectrograph. We cannot recommend shifting the collimator separately, for a change in the mutual arrangement of the parts of the spectrograph is often unsafe.

Let us say a few words on the selection of the width of the slit of a spectrograph. In contemporary spectrography we are forced to look for possible ways by which the light-gathering power of a spectrograph may be increased so that the spectra of the fainter stars might be obtained. The simplest way is to widen the slit. It is necessary to keep in mind that only in the rare cases of very good images is it possible to make use of the good optical qualities of a telescope. In most cases the visible disk of star in the focus of a telescope exceeds many times the diffraction disk (of the order of 0".10 - 0".20 in contemporary instruments of long focus) as well as the aberrational disc. (of the order of 0".5 - 0".8 in good instruments). As 1" at a focus of 16m corresponds to 0.077mm, it follows that for a width of 0.03 - 0.05 mm of the slit only a small part of the light is made use of. Experience shows that the exposure time decreases propprtionately to the width of the slit. If we take into consideration all the difficulties of the manufacturing of large instruments and add to it the fact that the gain in power by the increase of size is much smaller than is theoretically feasible, and that, besides, this gainappears only in good images, then it becomes clear that such a simple device for the increase of the power of the instrument as widening of the slit appears to be very desirable.

Unfortunately, the widening of the slit is connected with an increase in systematic and accidental errors (see § 12). Besides the purity of the spectrum decreases. The use of a wide slit increases the number of blended lines. In each case, in the selection of the width of the slit it is necessary to take into account the length of the collimator and of the camera. Besides, for the same instrument it is necessary to have a slit of different widths depending on the problem at hand. In the determination of radial velocities a width of 0.05mm may be used without lowering the accuracy, and in rare cases it may be even wider as for instance when the spectrum has many washed out lines or bands.

In photographing spectra of elongated objects (nebulae or comets, for instance) we may go further with respect to the width of the slit. Here we must differtiate between two cases: 1) when the spectrum consists of separate more or less monochromatic images and is dependent on the character of these radiations the widening of the slit is of little value after a certain limit is reached, for the same amount of light in a given line or band will spead over a correspondingly greater area; 2) when the spectrum is continuous, the slit should be widened as much as the character of the lines and bands in the spectrum permit (for instance, when the spectrum of faint nebulae is photographed the width of the slit is increased up to 0.2mm).

In the spectrographic investigations of lines it is desirable to use as narrow a slit as possible which is much narrower than the "natural" width of the narrowest lines. However, when we wish to investigate the total absorption in the lines and not contours we may use a wider slit.

§ 6. The Collimator. The objective of the collimator must satisfy high requirements with respect to spherical and chromatic aberrations. If the spherical aberration is noticeable and the various zones of the objective give a different focus, the parallelism of the pencil coming out from the collimator is destroyed thereby. This leads to a decrease of the purity of the spectrum. This effect influences somewhat differently the lines of absorption in the spectrum of stars and the lines of radiation from a voltaic arc, and it may lead to a systematic error in the determination of radial velocities of stars.

The objective of the collimator should be achromatized as much as possible. With a considerable chromatic aberration of the objective of the collimator it is impossible to obtain a good field in the spectrograph even if the objective of the camera is excellent in this respect. Since the slit of the collimator in stellar spectrographs occupies a very small part of the field near the center, no very high requirements are demanded of the objective of the collimator with respect to a good field.

A doublet or a triplet is selected nowadays as an objective of a collimator.

The walls of the tube of the collimator should be covered by diaphragms so that no rays reflected accidentally from the walls may fill on the objective of the collimator.

In order to utilize the entire light from a star it is necessary that the ratio of the diameter of the objective of the collimator to its focal distance be equal to the corresponding ratio of the telescope. For spectrographic work it is more advantageous to use a longer collimator. This, to be sure, requires a larger size of prisms which set a limit to the size of the collimator, for the preparation of good uniform prisms of large size is very difficult. The advantages of a long collimator are as follows: 1) With a long collimator the height of the prism is greater and the difference of the paths of the extreme rays is greater. Hence the resolving power of the prism

$$R = (t_1 - t_2)\frac{d\mu}{d\lambda}$$

and accordingly the purity of the spectrum is greater. 2) The width of the monochromatic image of the slit in the focal plane of the camera, if we neglect the phenomenon of diffraction is equal to

$$d\,\frac{F}{}$$

where d is the width of the slit, and f and f_1 are the focal distances of the objectives of the camera and collimator. Thus, other conditions being equal, longer collimators enable us to open the slit wider without decreasing the sharpness of the lines. This is connected with the increase of gathering power in the spectrograph. If the width of the slit remains the same, a longer collimator gives thinner lines. At present, that length of the collimator is selected which enables us to use prisms of two inches in height. This is the best compromise.

The focus of the collimator is usually determined for a long period of time. When a suitable selection of material (brass for instance) for the tube of the collimator with such a coefficient of widening which compensates the thermal change of the focal length of the objective of the collimator is made, the focus of the latter remains practically unchanged.

The determination of the focus of the collimator is done by Hartmann's method. The objective of the collimators, taken off the spectrograph, is placed on a supporting rod. A diaphragm with several symmetrically placed apertures is put on it. The slit is opened as wide as practicable, and a photographic plate is attached to it. In order to prevent any scratches on the slit, it is advantageous to cover the sides of the slit with thin paper and then press the plate to it. Taking two photographs on both sides of the focus, the exact position of the focus is determined by a known formula. It is desirable to use a blue filter while photographing in order to obtain sharper images. Besides, it is useful to know the chromatic curve of the objective of the collimator and it is determined by one of the several known methods. In mounting the slit at the focus it is necessary to take into consideration this chromatic curve. If the objective of the collimator is so achromatized that the vertex of the chromatic curve corresponds approximately to the same wave-length for which the prism is set at the angle of minimum deviation, it is desirable to take for the slit a focus which corresponds to the other points of the chromatic curve. In this case, two pencils of the corresponding wave-lengths symmetrically situated with respect to the central one will be upon emerging from the collimator, exactly parallel and therefore homocentric. The central pencil will also maintain its property of homocentricity because the prism is set in the position of minimum deviation for it. Only under these conditions will the rays after passing through the prism will be collected into one point again, and this is in essence of the property of homocentricity.

The application of this method enables us to obtain in focus a long portion of the spectrum.

§ 7. Prisms. Prisms used in spectrographs must satisfy exceptionally high requirements as to the uniformity and quality of their surfaces. If a prism is not homogeneous or if its surfaces have some curvature, it may act in part as a lens. As to the laboratory methods of testing prisms, they are expounded in detail in textbooks on optics.

There is, however, another method of testing a prism or a combination of prisms the spectrograph proper. For this purpose a series of diaphragms with rectangular cuts of 3 to 5mm in width and corresponding length are prepared. If we put them consecutively on the collimator with the cut placed parallel to the slit and to the refracting angle of the prism, these diaphragms will cut the pencils of rays which pass through various zones of a prism from vertex to

base. The difference of the foci of these zones will characterize the quality of the prisms. For the determination of the exact focus of the zones we may use Hartmann's method slightly changed. The photograph is taken on one side of the focus by using the diaphragm, put on the objective, passing the central zone; the slit diaphragm is so set that the middle of the slit illuminated by the voltaic arc is open. Next, on the same plate, but at another focus, is taken another photograph with a diaphragm cutting another zone; and the slit diaphragm is so shifted that the central part of the slit is closed, and a slit in two places symmetrically and closely placed with respect to the center. Thus, we shall obtain on the plate a spectrum corresponding to the central zone and two spectra corresponding to some other zone. Depending on the distance from the plate to the focus and on the real difference of the foci of the two zones, the spectra will be mutually shifted. This entire operation is repeated on another plate, and the plate holder with the plate is so placed that the plate is on the other side of the focus. Having measured the mutual shift of the spectra on both plates and knowing the mutual distance between the two positions of the plate holders, we obtain the position for the point of intersection of rays, corresponding to the central and the investigated zones, by the formula used in Hartmann's method for the investigation of objectives. Similar photographs are also taken for all the other zones. Introducing into the computations the corrections for the curvature of the spectral lines (see below §11) and relating them to a certain mean focal plane, we may easily find the deviations of the foci of the separate zones from vertex to base. This method is varied slightly depending on the structure of the spectrograph. In these tests it is useful to have the diaphragms with two cuts which are consecutively closed at the first and second photographing. Such a test may tell us much about the qualities of the prisms if we are certain that the objectives of the collimator and camera are free of defects. The existence of defective zones in the prism may be also discovered by a somewhat washed out character of the lines by the use of the proper diaphragms. Similarly, testing the prism by means of the same diaphragms situated so that their cuts be perpendicular to the refracting edge of the prism, we may obtain a notion as to the astigmatism of the entire system.

In order to identify that particular part of the spectrograph which causes the astigmatism, it is necessary to test the objectives of the camera and of the collimator separately.

It is necessary to make such a test before we begin any spectrographic observations. The existence of defects produces important consequences. Thus, if the surfaces of the prisms are not quite satisfactory, then, during a non-uniform illumination of the objective of the collimator (this often happens in astronomical practice) considerable errors are possible in the determination of radial velocities, and the purity of the spectrum also suffers (see below §12). As to astigmatism, its effect may be especially consequential whenever we deal with wide spectra as in the study of rotation of planets.

The problem of transparency of the prisms and their dimensions is important from the point of view of the effectiveness of the spectrograph. It was indicated above that the resolving power of a prism increases with the increase of its dimensions and that in modern spectrographs the best compromise is a height of about two inches. We must also decide here on the refracting angle base. The difference of the foci of these zones will characterize the quality of the prisms. For the determination of the exact focus of the zones we may use Hartmann's method slightly changed. The photograph is taken on one side of the focus by using the diaphragm, put on the objective, passing the central zone; the slit diaphragm is so set that the middle of the slit illuminated by the central zone is open. Next, on the same plate, but at another focus, is taken another photograph with a diaphragm passing another zone, and the slit diaphragm is so set that the central part of the slit is closed, and a slit in two places symmetrically and closely placed with respect to the center. Thus, we shall obtain on the plate a spectrum corresponding to the central zone and two spectra corresponding to some other zone. Depending on the distance from the plate to the focus and on the real difference of the foci of the two zones, the spectra will be mutually shifted. This entire operation is repeated on another plate, and the plate holder with the plate is so placed that the plate is on the other side of the focus. Having measured the mutual shift of the spectra on both plates and knowing the mutual distance between the two positions of the plate holders, we obtain the position for the point of intersection of rays, corresponding to the central and the investigated zones, by the formula used in Hartmann's method for the investigation of objectives. Similar photographs are also taken for all the other zones. Introducing into the computations the corrections for the curvature of the spectral lines (see below §11) and relating them to a certain mean focal plane, we may easily find the deviations of the foci of the separate zones from vertex to base. This method is varied slightly depending on the structure of the spectrograph. In some cases it is useful to have the diaphragms with two cuts which are consecutively closed at the first and second photographing. Such a test may tell us much about the qualities of the prisms if we are certain that the objectives of the collimator and camera are free of defects. The existence of defective zones in the prism may be also discovered by a somewhat washed out character of the lines by the use of the proper diaphragms. Similarly, testing the prism by means of the same diaphragms situated so that their cuts be perpendicular to the refracting edge of the prism, we may obtain a notion as to the astigmatism of the entire system.

In order to identify that particular part of the spectrograph which causes the astigmatism, it is necessary to test the objectives of the camera and of the collimator separately.

It is necessary to make such a test before we begin any spectrographic observations. The existence of defects produces important consequences. Thus, if the surfaces of the prisms are not quite satisfactory, then, during a non-uniform illumination of the objective of the collimator (this often happens in astronomical practice) considerable errors are possible in the determination of radial velocities, and the purity of the spectrum also suffers (see below § 12). As to astigmatism, its effect may be especially considerable when ever we deal with wide spectra as in the study of rotation of planets.

The problem of transparency of the prisms and their dimension is important from the point of view of the effectiveness of the spectrograph. It was indicated above that the resolving power of a prism increases with the increase of its dimensions and that in modern spectrographs the best compromise is a height of about two inches. We must also decide here on the refracting angle of a prism. A series of prisms of the same base but of different refracting angles have the same resolving power; however, in regard to losses in reflection and absorption they will not be equal.

The most advantageous compromise (especially in view of dispersion and loss of light by reflection) is a refracting angle of 60° in the prism.

As to losses due to absorption, it is necessary to pay serious attention to the sort of glass for a prism. At present, the Jena flint O.118 is the best, for, although it is not as good in regard to dispersion as some heavier sorts of flint, O.118 is more transparent especially for shorter wave lengths. This circumstance enables us to select the central ray of a shorter wave-length (let us say λ 4200) as compared with those in older spectrographs. This is of great advantage for the spectra of stars containing a small number of lines and situated in the more refracted part of the spectrum. In a spectrograph with a prism of this sort of glass the spectrum of stars may be followed up to 3700A.

Large prisms (see § 14) are often made of crown glass the angular dispersion of which is much less than that of flint glass.

Of great importance is the proper setting of the prism and its position with respect to the rays coming out from the objective of the collimator. The prism in its setting must be fastened firmly so that not even the slightest shift shall occur in the various positions of the telescope. On the other hand there should be no pressure on the prism because pressure may deform the prism and render it valueless. A prism set in its setting and fastened in a spectrograph should satisfy the following requirements: 1) the refracting edge should be parallel to the axis of rotation of the prism, and 2) the edges of a prism should be perpendicular to the optical axis of the camera and collimator. Usually, the mechanic who assembles the spectrograph takes care that these conditions be fulfilled. Finally, it is very essential that the pencil of light on passing through the prism shall remain homocentric. Only in this case will the image of the slit in the focal plane of the camera remain sharp. The homocentricity of the pencil of rays after its passage through the prism may be maintained under the two conditions: 1) when the thin pencil of rays passes through the prism under an angle of minimum deviation, and 2) when a pencil of parallel rays falls on the prism, and in this case the homocentricity is preserved also when the condition of minimum deviation is absent. This is the reason why a collimator is used. The pencil of rays of light coming out from the objective of the collimator may be regarded as complex of monochromatic parallel rays; upon the coming out of the prism we shall have a complex of monochromatic parallel rays of various directions, and this property is of great importance in the formation of the image at the focus of the camera. The combination of the parallal pencil of rays under the condition of least deviation helps greatly to obtain a sharp image along the length of the spectrum in the focal plane of the camera. The mounting of the prism at the angle of minimum deviation is done in the following manner: rotating the prism around the axis, we watch the shift, in the focal plane of the camera, of some image or a known spectral line. In the rotation of the prism the image moves first to one side then it stops and moves in the reverse direction. The position of the prism when the image stops corresponds to the angle of minimum deviation for a given wave-length. Usually such a mounting is made with respect to the central part of the spectrum.

We may say that the condition of minimum deviation is of much help toward the formation of a pure spectrum.

of a prism. A series of prisms of the same base but of different refracting angles have the same resolving power; however, in regard to losses in reflection and absorption they will not be equal.

The most advantageous compromise (especially in view of dispersion and loss of light by reflection) is a refracting angle of 60° in the prism.

As to losses due to absorption, it is necessary to pay serious attention to the sort of glass for a prism. At present, the Jena flint O.118 is the best, for, although it is not so good in regard to dispersion as some heavier sorts of flint, O.118 is more transparent especially for shorter wave lengths. This circumstance enables us to select the central ray of a shorter wave-length (let us say λ 4200) as compared with those in older spectrographs. This is of great advantage for the spectra of stars containing a small number of lines and situated in the more refracted part of the spectrum. In a spectrograph with a prism of this sort of glass the spectrum of stars may be followed up to 3700A.

Large prisms (see § 14) are often made of crown glass the angular dispersion of which is much less than that of flint glass.

Of great importance is the proper setting of the prism and its position with respect to the rays coming out from the objective of the collimator. The prism in its setting must be fastened firmly so that not even the slightest shift shall occur in the various positions of the telescope. On the other hand there should be no pressure on the prism because pressure may deform the prism and render it valueless. A prism set in its setting and fastened in a spectrograph should satisfy the following requirements: 1) the refracting edge should be parallel to the axis of rotation of the prism, and 2) the edges of a prism should be perpendicular to the optical axis of the camera and collimator. Usually, the mechanic who assembles the spectrograph takes care that these conditions be fulfilled. Finally, it is very essential that the pencil of light on passing through the prism shall remain homocentric. Only in this case will the image of the slit in the focal plane of the camera remain sharp. The homocentricity of the pencil of rays after its passage through the prism may be maintained under the two conditions: 1) when the thin pencil of rays passes through the prism under an angle of minimum deviation, and 2) when a pencil of parallel rays falls on the prism, and in this case the homocentricity is preserved also when the condition of minimum deviation is absent. This is the reason why a collimator is used. The pencil of rays of light coming out from the objective of the collimator may be regarded as complex of monochromatic parallel rays, upon the coming out of the prism we shall have a complex of monochromatic parallel rays of various directions, and this property is of great importance in the formation of the image at the focus of the camera. The combination of the parallel pencil of rays under the condition of least deviation helps greatly to obtain a sharp image along the length of the spectrum in the focal plane of the camera. The mounting of the prism at the angle of minimum deviation is done in the following manner: rotating the prism around the axis, we watch the slit, in the focal plane of the camera, or some image of a known spectral line. In the rotation of the prism the image moves first to one side then it stops and moves in the opposite direction. The position of the prism when the image stops corresponds to the angle of minimum deviation for a given wave-length. Usually such a mounting is made with respect to the central part of the spectrum.

We may say that the condition of minimum deviation is of much help toward the formation of a pure spectrum.

Next, when the prism is in a position corresponding to the angle of minimum deviation, small shifts of the prism at various zenith distances and hour angles produce the least effect (shift of spectrum) in the determination of radial velocities as compared with what takes place at any other position of the prism. Finally, at the angle of minimum deviation there is the least loss of light by reflection of light from the prism.

§ 8. The Camera. The objective of the camera is made somewhat larger than that of the collimator as the pencil of light coming out from the prism is somewhat wider than that entering it. Usually, the spectrograph has several cameras the focal distances of which determine the linear dispersion for a given number of prisms. The camera objectives used in stellar spectrography usually have a limited good field of the order of 2° - 3°. Usually for a spectrograph of three prisms in good focus the portion of the spectrum obtained is merely 200 to 400 Å long which fact is a great limitation in the work on radial velocities and especially in photometric investigations of spectral lines.

Of late, the problem of obtaining a longer portion of the spectrum, and sometimes of the entire visible portion of the spectrum, may be considered as solved. The first new interesting innovation in this field is Zeiss's "chromat". It is an objective made of the same sort of glass as that of the prism and it consists of two menisci, one positive and the other negative, separated by a layer of air. The chromatic aberration is not corrected here because both lenses are made of the same material. However by a suitable selection of the inclination of the plate holder it is possible to obtain a good focus along a large portion of the spectrum. With such an objective the field is widened by about 15° Zeiss's "chromat" yields excellent results only with long cameras having a ratio of more than 1:12.

The other serious deficiency of this objective is the considerable inclination of the plate holder, for the slightest deviation from the focus causes a considerable error (§12).

For a spectrograph of a single prism it is possible to obtain an excellent field by taking as an objective of the camera the Hastings-Brashear triplet. Having a light gathering power of 1:12 and the inclination of the plate holder at 2°, the entire spectrum from 3800 to 6700Å will be in good focus and the extreme deviation will not exceed 0.1mm. There are several types of Hastings-Brashear objectives. However, the getting of flat field is rendered more difficult if three prisms are used at cameras of very short focus. In general, the character of the focal surface of the spectrograph depends chiefly on the following factors: 1) on the chromatic curve of the objective of the collimator, 2) on the chromatic curve of the objective of the camera, 3) on the curvature of the field of the objective of the camera, and 4) on the degree of deviation from the condition of homocentricity for the rays of different wave-lengths. The farther the rays of a given wave-length are from the middle of the spectrum and the greater is the dispersion, the more difficult it is to obtain a flat field for long portion of the spectrum. In spectrographs of a single prism at a moderate dispersion an excellent chromatic curve is often obtained.

This curve is represented on figure 20, page 131 for the single - prism spectrograph used together with the 40 inch reflector on the Simeis observatory.

At present spectrographs of greater light-gathering power are becoming of greater importance because in the determination of radial velocities, the bright stars are already covered and work must be done on the fainter stars among which are certain stars of the greatest ~~astro~~astrophysical interest. Into astronomical practice are now introduced cameras of short focus and of large light-gathering power. They enable us to gain in brightness at the expense of loss in linear dispersion and in purity of the spectrum. The gain in brightness may be justified the more because certain faint objects produce spectra with washed out, wide and irregular lines so that the decrease in linear dispersion has little effect on the accuracy of the determination of radial velocities.

However, the difficulty of obtaining flat field increases with the increase of the angular aperture. For objectives of short focal length an excellent field is required and a shortening of 0.1 mm with a light-gathering power of 1:5 already affects the quality of the lines in the spectrum. Those spectrographs of large light-gathering power which produce a small linear dispersion and which consist of two or three prisms in combination with cameras of very short focus are very advargageous. Since the resolving power with the same length of base of prism depends on the number of prisms, such a combination enables us to obtain with a small linear dispersion, a sufficiently pure spectrum and a considerable gain in brightness. The gain in brightness may become still greater because the slit in a camera of very short focus may be opened wider. For such spectrographs of great light-gathering power the most advantageous objectives are those of Moffit (representing a modification of objectives of the Petzvaal type), which produce very good results with a light-gathering power of 1:5 (the flat field is within the limits of 0.05). The flat field and the sharp images are also given by the Tessar Zeiss and Ross's Special Homocentric objectives.

Ross's objectives may go farther with respect to light-gathering power, but the Tessar gives a flatter field and the plate remains perpendicular to the optical axis.

Spectrographs of maximum light-gathering power are required for the spectrographic investigations of nebulae. This is connected with the fact that the nebulae are very faint, where as an increase of the size of the telescope offers no advantages to elongated celestial objects. Of late complicated objectives of microscopes (Righton's camera with a ratio of 1:1 or even 1:0.7) are introduced into practice for this purpose, so that the plate almost touches the rear surface of the objective,

By means of such spectrographs spectra of nebulae of 16-th magnitude are photographed at Mount Wilson.

When large dispersion and purity of spectrum are required, three prisms are usually used. But in this case all objectives give a field concave toward the objective so that only a limited portion of the spectrum may be photographed at once. Although Zeiss's "Chromat" and the Brashear Light Crown give a satisfactory result, their use is connected with a considerable inclination of the plate holder (especially for Zeiss's "Chromat") which is a shortcoming in the determination of radial velocities and in spectrographic investigations.

In this respect the best objectives are the contemporary astronomical objectives of Ross (of four lenses) which give a remarkably flat field of 15° in diameter. With these objectives, specially designed for spectrographs, excellent results are obtained in respect to field and sharpness of images. These results could not be obtained by objectives of other types. The field may be made of still greater use if the plate in the casket is given a certain bending and corresponding springs are set in the casket. These objectives are especially useful in work on the photometry of spectral lines, as, in this case, a large dispersion is possible and yet it desirable to obtain immediately, in good focus, the maximum length of a portion of the spectrum.

§ 9. The Mechanical Construction of the Spectrograph and the Flexure.

The laboratory universal type was taken as the base of construction of the stellar spectrograph. It soon became clear that special changes in construction in which the inevitable flexure of the spectrograph, as of a certain flexible system which does not remain at rest, must be taken into consideration. This is especially important if we determine radial velocities of stars when the exposure often lasts scores of minutes and even several hours.

From the point of view of the mechanical construction of spectrographs (in connection with the effect of flexure) two types are to be distinguished. In the first type the spectrograph alone, or the spectrograph with its supporting system and with which it constitutes a whole, is fastened directly to the telescope. It is natural to expect in this case that the spectrograph having one or several fulcrums in the place where it is fastened to the tuble will suffer a large differention flexures similiar to a beam fastened to a wall by one of its ends. In spectrographs of this type, which were used at first, the effect of flexure was so large during a long exposure that it was impossible to determine the radial velocities. Even with an exposure of one hour the shift of the spectral lines as a result of the differential flexure produced an error in radial velocities in many kilometers.

The first improvement was a firmer fastening of the various parts of the spectrograph to each other.

In the second type of spectrograph the influence of the differential flexure on the measured radial velocity may be reduced to only a small fraction of a kilometer. This is done by the application of a new principle, proposed by Wright, consisting in the spectrograph and its supporting system being constructed as two independent parts. The so called "support", the system supporting the spectrograph, is directly fastened to the telescope, and this "support" is made as rigid as possible. The spectrograph proper rests on this system to which it is bound at two points properly selected. The position of these supporting points may be seen in figure 21 and 22.

One of these supporting parts, which is usually between the prism and the camera, is an axis passing through the spectrograph. The other support is on the upper part of the spectrograph. In some constructions the upper support serves chiefly to prevent the spectrograph from rotating around the lower axis. From the ends of the lower axis(and sometimes also from the upper) are projected braces which connect this supporting system with the telescope. The connections of the spectrograph with its supporting system is such that the flexure of the separate parts of the supporting system will not cause harmful strains in the separate parts of the spectrograph. With the suitable selection of the fulcrums (which may be calculated theoretically on the bases of the weight and sizes of the separate parts of the spectrograph, or on the bases of test experiments) it is possible to distribute the flexure between the supporting points and outside of them in such a way that the effect of flexure will be very small. Besides, the entire system is made very rigid and firm. In this case in a two hour exposure the shift of the spectrum because of the differential flexure may be reduced to a very small magnitude, in linear measure to

less than 0.0005mm. Since, in addition, the comparison spectrum of the terrestrial source is exposed at least two times (at the beginning and at the end of the exposure of the star), it appears that for a spectrograph of a single prism the influence of flexure on the radial velocity will be only a fraction of 0.0005mm. Expressed in kilometers, the effect of flexure in the determination of radial velocities may be reduced to 0.2 to 0.3 kilometers.

The progress in the decrease of flexure was obtained chiefly by the separation of the spectrograph proper from its supporting system with which it is connected schematically at two points. The considerable flexure compelled the abandonment of the original universal type in which it was possible to change dispersions by changing the number of prisms. The division of the spectrographs into those of one prism, of two prisms, and of three prisms enabled us to make them more compact. Of late, however, the universal type of the compact sort is again being introduced (at Victoria), but maintaining the principle of attaching the spectrograph proper to its supporting system (fig. 23).

It is necessary to distinguish between general and differential flexure. In the determination of radial velocities the most dangerous effect is that of the latter, which shows itself during the exposure in changing the position of the plate holder part of the camera with respect to the other parts of the spectrograph, especially with respect to the slit.

The general flexure produces an effect here of secondary order only. Thus, when the optical axes of the telescope and spectrograph are brought into complete coincidence at the given position of the spectrograph, this coincidence is destroyed at any other position because of the general flexure. Therefore an uneven illumination of the objective of the collimator takes place which fact may cause a systematic error in the determination of radial velocities (§12). The effect of the general flexure of the spectrograph is very difficult to separate from the effect of the general flexure of the tube (§12).

It is also necessary to note that the same flexure will be of greater effect in a spectrograph of a single prism as compared with that of a spectrograph of three prisms. This is explained, on one hand, by the difference in scales or linear dispersion (since the same angular change causes a linear shift which is inversly proportional to the dispersion), and, on the other hand, by the difference of the mutual position of the separate parts in spectrographs of a single prism and that of spectrographs of three prisms. In the latter case the angle of deviation is close to 180°, the slit and the plate holder may be very close to each other which fact decreases the effect of flexure on the contrary in a spectrograph of a single prism the slit and the prism are at maximum distance from each other and therefore the effect of flexure will be greater.

The real shift of spectral lines because of flexure follows a quite complicated law the determination of which before hand is difficult to make. The effect of flexure depends on whether the collimator is on the prolongation of the geometric axis of the telescope or is perpendicular to it. The flexure also depends on the mutual position of the separate parts of the spectrograph, their weight, and the angle of deviation of prisms.

Nevertheless, the flexure may be expressed by some formula. If F is the maximum flexure when the telescope is directed towards the southern point of the horizon (which is true of most spectrographs of three prisms), then, at any other position of the telescope and spectrograph, the flexure f(which is the component influencing the radial velocity) may be expressed approximately by the following formula:

$$f = F \sin z \cos p = F(\cos\delta \sin\varphi - \sin\delta \cos\varphi \cos t) \quad (18)$$

where δ is the declination, z is the zenith distance, p is the parallactic angle, is the latitude of the place, t is the hour angle.

Hence it follows that the differential flexure during the exposure $t_2 - t_1$, is equal to

$$\Delta f = F \sin\delta \cos\varphi (\cos t_2 - \cos t_1) \quad (19)$$

In a more general case (especially for spectrographs of a single prism) the minimum of flexure obtains not when the telescope is directed towards the zenith but at a certain zenith distance z. If we again denote by F the maximum flexure (the point at a distance of 90° from z), we shall similarly obtain

$$\Delta f = F \sin(\delta - z) \cos\varphi (\cos t_2 - \cos t_1). \quad (20)$$

The formulae (19) and (20) are quite approximate. The effect of flexure is usually determined in an experimental way. First the telescope is directed to any point on the meridian, and the comparison spectrum Fe and Ti is exposed. Next, without touching the plate holder the telescope is directed toward another distant point on the meridian, or, without changing the declination, toward a point at a large hour angle west ward or east ward the comparison spectrum is exposed again on the same plate (the diaphragm on slit is so shifted that both spectra appear side by side). Measuring the shift of the spectra in a series of such pairs of photographs it is possible to determine the value of the maximum flexure and its corresponding value of the zenith distance.

10. The Mounting of a Spectrograph. The separate parts of a spectrograph as well as the spectrograph as a whole were described in the previous sections. Now we shall pass on to the description of the basic operations necessary before we can start regular observations with the spectrograph. First it is necessary to assure ourselves that the separate optical parts of the telescope proper are well centered with respect to one another. This is done by means of well known methods. After that the spectrograph is attached to the ocular end of the telescope. Next it is necessary to bring into coincidence the axis of the collimator and optical axis of the telescope. The conditions of centering vary depending or whether the spectrograph is situated on the prolongation of the geometric axis of the telescope or is situated perpendicularly to this axis. Essentially, it is necessary to get such a state of affairs that the pencil of light, entering through the slit into the collimator, illuminates the objective of the collimator uniformly and entirely, i.e. be concentric with respect to it. This is obtained in the following manner. The mirror or the objective of the telescope are greatly diaphragmed, and it is desirable to make such a diaphragm which has at its

edge a series of apertures of 1 to 2 cm situated concentrically with respect to the mirror or the objective of the telescope. On the objective of the collimator is put a ring with a lusterless plate with concentric circles. The telescope is directed towards the Sun and, setting the corresponding diaphragm on the slit so that a small, almost square, aperture appears, a circle of small images is obtained on the ground glass. This circle of images should be concentric with respect to the edge of the objective of the collimator. This is done by the aid of set screws and lock - nuts by which the spectrograph is fastened to the telescope. In the case of a reflector of the Cassegrainian type, when the large mirror has no cut in the central part, the concentricity of images on the ground glass is obtained, in part, by a slight regulation of the third flat small mirror of the reflector. This centering should be made at a very small zenith distance. If possible it is better to use the illumination from the sky and not from the Sun. Sometimes, for spectrographic observations instruments are used, the objective of which is achromatized for visual rays. In this case the correcting lense is at first centered separately with respect to the objective of the collimator. It is of great importance to obtain a complete and uniform illumination of the objective of the collimator. Another way of assuring ourselves of the correctness of the centering is the following. A bright star is accurately held on the slit, and a photographic plate is attached to the objective of the collimator. If the plate is uniformly darkened and if the bright little circle (in the case of a reflector), corresponding to the diaphragming small mirrors, is exactly in the center, then the centering is quite accurate. Otherwise it is still to be attained.

The focusing of the complete spectrograph the parts of which are already regulated is done quickly and accurately according to Hartmann's method (somewhat changed) for the investigation of objectives. On the objective of the collimator or camera is put a semicircular diaphragm which, upon rotation through 180°, enables the pencil of light to pass through the upper or lower half. On the same plate are obtained side by side two comparison spectra at one and then at the other position of the semicircular diaphragm (in taking the second photograph the diaphragm on the slit is correspondingly shifted). If the plate is not at the focus, the lines in both spectra will be mutually shifted, and, as can be easily comprehended, the shift of the lines will be different directions depending on whether the plate is on one or the other side of the focus. Having obtained 4 or 5 such plates at different settings of the focusing scale it is possible to determine the focus easily and quickly, without any measurements, with an accuracy up to 0.05mm by observing the spectrogram with a magnifying glass as to the shift of lines. The only deficiency in this method is that in case the prism is not quite homogeneous (§ 12) the determination of the focus will include a systematic error.

In order to obtain as long a portion of the spectrum as possible in good focus it is necessary to incline the plate holder part for most cameras. The correct inclination of the plate holder is determined by the same method by which the focus is determined. It is assumed here that the plate holder part can be rotated around an axis passing aproximately near the middle of the plate holder. First, the focus for the middle part of plate is determined. Next, two comparison spectra corresponding to both positions of the semicircular diaphragm placed on the objective of the collimator or camera and

opening the upper and lower halves, are set side by side and observed through a magnifying glass. If the mutual shift of the spectral lines of both comparison spectra in the extreme regions of the plate is directed towards different sides, it shows that both ends of the plate are on different sides of the focus and therefore the inclination should be corrected. This operation should be continued (each time correcting the focus of the central part of the spectrum) until in the extreme parts of the spectrum a mutual shift of lines in the smae direction by the same amount will take place. This will show that the inclination is correct and that the deviation from the focus depends on the curvature of the field or on the deviation of the chromatic curve from a plane.

3. Determination of Radial Velocities.

§11. **Measurements and Reductions of Spectrograms.** The measurements of spectrograms is usually made by means of a microscope having a measuring screw. Essential parts of this instrument are the micrometer screw, by means of which it is possible to move the holder of the spectrogram, and a good microscope of small or average magnification. The cross-hairs at the end of the eye-glass, the upper part of the spectrogram-holder or carriage and the spectrogram itself may be so placed that the cross hairs always remain perpendicular to the spectrum, and the direction along the length of the spectrum remains parallel to the motion of the carriage.

During the measurements the readings are usually made by means of two indices which are situated side by side and which are divided by a thin plate with a marked scale. One index shows complete revolutions of the screw, the other only portions of a revolution. It is desirable to have at hand two or more objectives, so that the magnification of the microscope can be changed in accordance with the nature of the spectrum. For spectra of thin and sharp lines the magnification should be from 20 to 30 times, and in measuring spectra of washed out and very faint lines it is preferable to magnify from 10 to 15 times. Besides, the magnification may be changed according to the granulation of the plate. Plates which are less sensitive and of smaller grain in their granulation permit a larger magnification. To be sure, the spectrogram must be well and uniformly lighted by means of a mirror.

The errors made during the measurements of spectrograms may be various. First, it is necessary to investigate the periodic and progressive error of the screw, with a large magnification by means of the method well known in practical astronomy. In modern micrometric screws these errors are very small (of the order of the fourth decimal) and are not taken into consideration any more. If the error is noticeable (of the order of the second or third decimal), tables of corresponding corrections are made and posted. However, for the determination of radial velocities (the mean of a large number of lines) the influence of this error will be very small.

Another source of error is the systematic difference of setting upon the bright lines in the spectrum of a star and upon the dark lines in the comparison spectrum (we should remember that the negatives of the spectra are measured). If the spectrogram is measured in its two positions differing by 180° from each other, the mean value of the two measurements will be largely free of this error.

Next, the influence of the temperature may show in the measurements. It is very desirable that the temperature remain as constant as possible during the measurements; besides, the measurements should not be interrupted. In uninterrupted measurements the influence of the change of temperature on the screw and other parts of the insturment is much less. It is necessary to keep in mind the possibility of accidental errors of setting depending on the quality of the lines and the character of the plates. If the lines in the spectrum are very good, the accuracy of setting on the stellar spectra is better than that of the comparison spectra. This is because the setting of a dark cross-hair on a dark thin line of the artificial spectrum of F_e or F_i is less accurate than the setting upon a bright line of a stellar spectrum. But, on the average, because the lines in a stellar spectrum are rarely isolated, sharp, and symmetrical, the accuracy of setting in both case becomes the same, or it even becomes more accurate for lines of the comparison spectra.

The purpose of measuring a spectrogram is to determine the relative wave-lengths of the spectral lines. The spectrogram is so situated on the instrument that the reading on a selected line (about the middle) is always the same. Then, the sum of readings of the two positions (the violet end on the right, and then violet end on the left) for all lines will be about the same and it will be close to a whole number, let us say, 180 or 200. Subtracting from this number all the readings in the second position, we shall obtain values for separate lines close to the readings of the first position of the spectrogram. It is desirable that the readings at the first position should increase as the wave-lengths increase. Taking the mean values of the readings in the first position and in the reversed second position, we shall obtain results which will be free to a large extent from errors of setting which depend on the character and form of the various lines. As we said before, by a turn of 180° the error, which depends on the differences of setting on bright stellar lines and on dark lines in the comparison spectrum, will be eliminated.

If we depict, on millimeter paper, the wave-lengths as abscissae and the final means readings as ordinates, the points will settle on a certain curve which is close to a hyperbola. Cornu and Hartmann found that these curves may be more or less (depending on the magnitude of the measured portion of the spectrum) represented by the formula

$$\lambda - \lambda_0 = \frac{C}{x_0 - x} \tag{21}$$

or by

$$\lambda - \lambda_0 = \frac{C}{(x_0 - x)^{\alpha}} \tag{22}$$

In these dispersion formulae λ_0, x_0 and C are constants, λ is the wave-length, α in (22) is usually . The second formula is more useful for longer portions of the spectrum. The application of these formulae is as follows: readings are obtained for a series of lines, in the comparison spectrum, of well known wave-lengths; by three of these lines (one in the middle and one on

each end) the constants are determined by the formulae (21) and (22); and conversely, by these constants and by the same formulae x is computed for all other lines of the comparison spectrum. Comparing the results with the measured values of x it is possible to know how well the formula represents the measurement. By means of corrections obtained for the comparison spectrum it is possible to correct the readings of the stellar lines and find corresponding values of wave-lengths for them by the formulae (21) and (22).

The constants of the formula are easily determined. It is desirable to introduce in the solution the following new unknowns instead of x_1, x_2, x_3.

$$x'_1 = x_1 - x_2, \quad x'_2 = 0 \quad \text{and} \quad x'_3 = x_3 - x_2.$$

Then the constants are determined by the following formulae

$$\lambda_0 = \frac{\frac{x'_1}{x'_3}(\lambda_2 - \lambda_3)\lambda_1 - (\lambda_2 - \lambda_1)\lambda_3}{\frac{x'_1}{x'_3}(\lambda_2 - \lambda_3) - (\lambda_2 - \lambda_1)} \qquad (23)$$

$$x_0 = \frac{x'_1}{\frac{\lambda_2 - \lambda_0}{\lambda_1 - \lambda_0} - 1} = \frac{x'_3}{\frac{\lambda_2 - \lambda_0}{\lambda_3 - \lambda_0} - 1} \qquad (24)$$

$$C = (\lambda_1 - \lambda_0)(x_0 - x_1) = (\lambda_2 - \lambda_0)(x_0 - x_2) = (\lambda_3 - \lambda_0)(x_0 - x_3) \qquad (25)$$

Of late, formula (21) is mostly used. Only for longer portions of the spectrum (22) is applied. The constants are computed by means of successive approximations for a series of values 0.5, 0.6 etc. In the formula

$$\frac{x'_1}{x'_3} = \frac{\left(\frac{\lambda_2 - \lambda_0}{\lambda_1 - \lambda_0}\right)^{\alpha} - 1}{\left(\frac{\lambda_2 - \lambda_0}{\lambda_3 - \lambda_0}\right)^{\alpha} - 1} \qquad (26$$

λ_0 is so selected that for an assigned α (often α = 0.5) the computed $\frac{x'_1}{x'_3}$ coincides with the measured value. Then X_0 is computed for λ_0 by formula

$$x_0 = \frac{-x'_1}{\left(\frac{\lambda_3 - \lambda_0}{\lambda_1 - \lambda_0}\right)^{\alpha} - 1} = \frac{-x'_3}{\left(\frac{\lambda_2 - \lambda_0}{\lambda_3 - \lambda_0}\right)^{\alpha} - 1} \qquad (27$$

The three values of C, which are to coincide, are computed by formula (22). In practical application formula (21) is substituted by formula

$$x_0 - x = \frac{C}{\lambda + \Delta\lambda - \lambda_0} \text{ or } \lambda - \lambda_0 = \frac{C}{x_0 - x - \Delta x}, \quad (28)$$

where Δx and $\Delta\lambda$ are empirical corrections obtained according to the lines of the comparison spectrum as was explained above. Formula (22) is substituted likewise.

In mass determinations of radial velocities it is impractical to compute the dispersion formula for each star. Even if we use formula (21), the reduction of a spectrogram takes much time (the computations are carried out with tables of logarithms of six or seven decimal places). Instead, a simple method is used which is a simplification of Hartmann's method. For the same spectrograph having a given dispersion the constants change within very small limits, chiefly because of temperature changes. If we compare two spectrograms, with the comparison spectrum, it will appear that their dispersion is different. The readings for the same lines on one of them will differ systematically from the other. If for a certain middle line the readings coincide, they will increasingly differ on both sides of this line. These deviations, derived from the comparisons of the readings for the lines of the comparison spectrum, enable us to reduce all the readings of the second spectrogram to that dispersion which obtains in the first spectrogram.

Using this principle, 8 or 10 spectrograms obtained at the same temperature are measured with the comparison spectrum, and the mean readings are taken which are thus quite accurate. By these readings the constants of the basic dispersion formula (28) are computed. The latter serves for the computation of tables of "normal" readings for all lines is the spectra of stars for which the wavelengths are known. The table of the computed "normal" readings for the lines of the comparison spectrum and the corresponding table of computed "normal" readings (for the same dispersion) of stellar lines are the bases for further reductions of all obtained spectrograms.

The obtained readings for the lines of the comparison spectrum on the given stellar spectrogram will differ from the "normal" readings. The differences between them are recorded on graph paper as a function of the wavelength and a curve is drawn. Using the curve, corrections are found for the stellar lines. The corrections for the lines of the comparison spectrum is made of two parts: 1) a constant part due to the fact that during the measurement of a given spectrogram, a certain initial reading of the line of the comparison spectrum (usually in the middle of the spectrum) has accidentally not coincided exactly with the "normal" reading for the same line, and 2) a variable part depending on how much the dispersion has changed; the variable part increases as the distance from the initial reading increases.

The signs of the corrections should be such that, when the corrections are added to the measured readings of the lines of the comparison spectrum, the readings are "normal". The correspondingly corrected readings of the stellar lines are compared with the normal readings for them and the obtained deviation should be regarded as a Doppler shift.

It is necessary to keep in mind that, if the temperature, at which a given spectrogram was obtained differs from that at which the basic spectrogram with "normal" readings were obtained, the corrections for the reduction

to "normal" dispersion will be very large. Therefore, it is necessary to obtain a series of dispersion formulae for various temperatures. Their constants are evened up somewhat in order to avoid accidental errors, and by them several tables of normal readings, let us say for each 10°, are computed. In this case "normal" readings corresponding to the closest temperature are used for the reduction of a stellar spectrogram.

The obtained shifts of stellar lines (corrected readings minus the "normal" ones), given in fractions of a revolution of the screw, should be transformed into kilometers. For this purpose a table is used which gives the number of kilometers corresponding to a single revolution of the screw for various wave-lengths. In order to compute this table formula (21) should be differentiated

$$dx = \frac{C\,d\lambda}{(\lambda-\lambda_0)^2}.$$

Substituting Doppler's formula

$$d\lambda = \frac{v}{C}\lambda,$$

where c = 299860 km/sec, we find

$$v = \frac{299860}{\lambda}\frac{(\lambda-\lambda_0)^2}{C}dx,$$

where $\frac{299860}{\lambda}\cdot\frac{(\lambda-\lambda_0)^2}{C}$ is the conversion factor for which the table is made. Since this conversion factor changes somewhat with the dispersion, a corresponding correction is needed for very accurate measurements.

The entire process of reduction is very simple and takes little time. We shall give here an example for the sake of illustration (the 6-th column is dropped during the process)

The large amplitude of velocities in the last column is due, in part, to the errors of the accepted values of wave-lengths of stellar lines in the first column.

Of the other methods of reducing spectrograms Schlesinger's method may be mentioned. It is a modification of the method described above. In this case the table of "normal" readings for the stellar lines and for the comparison spectrum are computed first, (the same tables as in Hartmann's method). The method is based on the assumption that on very small portions of the spectrum the change of dispersion from plate to plate can be represented by a linear formula. Let R_1 and R_3 be the normal readings for lines of iron and let R_2 be the normal reading between R_1 and R_3 for the lines of a star. Next, let r_1, r_2, r_3 respectively be the readings for the given spectrogram. Were the dispersion the same for both spectrograms, the magnitude

$$(r_2 - r_1) - (R_2 - R_1)$$

Table 5

Simeis. A Spectrograph of one prism. 110 Herculis. 1931 September 3, 688

T	First Reading I	Second reading II	$\frac{I+180-II}{2}$
4021.87 Fe	69.539	110.468	69.536
4045.79 *	71.377	108.641	71.368
4063.57 *	72.668	107.338	72.665
4076.64 Fe	73.554	106.452	73.551
4143.69 *	78.220	101.790	78.215
4147.68 Fe	78.448	101.556	78.446
4198.51 *	81.799	98.220	81.790
4202.02 *	82.022	97.992	82.015
4210.36 Fe	82.504	97.508	82.498
4215.58 *	82.856	97.156	82.850
4226.95 *	83.562	96.446	83.559
4233.26 *	83.957	96.058	83.950
4235.84 *	84.120	95.822	84.114
4250.46 *	84.998	95.020	84.989
4271.52 *	86.272	93.746	86.263
4282.41 Fe	86.872	93.138	86.867
4289.94 *	87.347	92.680	87.334
4325.76 *	89.398	90.622	89.388
4337.05 Fe	89.998	90.009	89.994
4340.47 *	90.226	89.794	90.216
4351.83 *	90.858	89.166	90.846
4375.93 Fe	92.136	87.873	92.132
4383.55 *	92.573	87.446	92.564
4395.12 *	93.188	86.830	93.179
4404.71 *	93.704	86.306	93.699
4447.72 Fe	95.890	84.114	95.888
4481.23 *	97.606	82.430	97.588
4494.57 Fe	98.225	81.784	98.220
4549.53 *	100.880	79.136	100.872
4602.95 Fe	103.287	76.720	103.284

Table 5

Simeis. A Spectrograph of one prism. 110 Herculis. 1931 September 3, 688

Continued from the other page

Reduction to Normal Dispersion	Normal Reading	Shift in Revolutions	Shift in Km/Sec
+ 0.089	69.625		
	71.399	+ 0.049	+ 50
	72.688	+ .051	+ 52
+ .072	73.623		
	78.232	+ .032	+ 35
+ .048	78.494		
	81.785	+ .041	+ 46
	82.007	+ .042	+ 47
+ .031	82.529		
	82.854	+ .027	+ 31
	83.558	+ .029	+ 33
	83.944	+ .033	+ 38
	84.103	+ .037	+ 43
	84.988	+ .024	+ 28
	86.243	+ .038	+ 45
+ .015	86.882		
	87.321	+ .027	+ 33
	89.368	+ .027	+ 33
+ .006	90.000		
	90.189	+ .031	+ 38
	90.816	+ .032	+ 40
- .003	92.129		
	92.537	+ .023	+ 29
	93.151	+ .022	+ 28
	93.657	+ .034	+ 44
- .015	95.873		
	97.545	+ .023	+ 31
- .021	98.199		
	100.813	+ .024	+ 41
- .041	103.243		
			+ 38.2

would have evidently represented the shift of lines caused by Doppler's effect. In reality the dispersion is different and its change on the portion between R_1 and R_2 is equal to

$$(r_3 - r_1) - (R_3 - R_1).$$

Let us denote by K the ratio

$$K = \frac{R_2 - R_1}{R_3 - R_1}$$

If the change of dispersion on the portion $R_3 - R_1$ can be represented by a linear formula, the change on the portion $R_2 - R_1$ will be equal to

$$K\left[(r_3 - r_1) - (R_3 - R_1)\right]$$

This correction with an opposite sign should be added to the quantity $\left[(r_2 - r_1) - (R_2 - R_1)\right]$ in order to obtain the shift of the stellar line caused by the motion along the line of sight, i.e.

$$\Delta x = (r_2 - r_1) - (R_2 - R_1) - k\left[(r_3 - r_1) - (R_3 - R_1)\right] \quad (31)$$

In spite of the cumbersome appearance of this formula, the computations are swift, as the quantities $R_2 - R_1$, $R_3 - R_1$, and K are already tabulated for the normal readings.

If we select a line of the comparison spectrum so that the difference $R_3 - R_1$ is not to exceed five revolutions of the screw, the linear formula for the change of dispersion will be sufficiently accurate. It is easy to see that in Schlesinger's method the reduction curve is replaced by a broken line consisting of a certain number of linear sections $R_3 - R_1$. In the first method an interpolational curve passing through the vertices of this broken line is taken.

The deficiency in Schlesinger's method is that the error of setting on iron lines (especially on R_1) is entirely transfered into the shift of the stellar line. On the contrary, its advantage for a small number of lines in a stellar spectrum is the swiftness of reduction without any graphic constructions. In swiftness of reduction the methods of Hartmann and of Schlesinger are equal.

In the measurement of radial velocities it is necessary to keep in mind that for a rectilinear slit the spectral line appears in the form of a curve with its convex side turned towards the red end of the spectrum. This curvature is caused and depends on the fact that the angles of incidence of rays coming out from the middle of the slit and those coming out from the left and right ends will not be equal, and hence the passage of the rays in the prism will not be the same. As the angle of refraction depends also on the index of refraction which increased toward the violet end, the difference in the passage of the rays from the center of the slit and from the left and right ends will increase in the same direction, and the effect of curvature

will be greater for shorter wave-lengths.

In order to determine the corrections for the curvature of lines the iron spectrum is photographed at as long a slit as possible. The figure of the line may be represented by the parabola $x = ay^2$. Measuring x and y in different points of the line we determine the constant . Next, by argument y (which is equal to half of the distance between points of the comparison spectrum on which the cross-hairs are set during measurement) we compute the correction x. The correction thus computed will be quite accurate if the setting on the line of the comparison spectrum is symmetrical with respect to the lines of the stellar spectrum. Otherwise, the reduction will be somewhat more complicated. Such a correction is determined for various portions of the spectrum. Correction x, translated into kilometers, is added with a negative sign to the velocity of stars. If the spectrum of a star is narrow and the ends of the lines of the comparison spectrum approach closely the spectrum of the star, the correction then becomes very small. The corrections for a spectrograph of a single prism are in general very small consisting mostly of 0.2 to 0.3 km. Since with the increases, the correction may reach several kilometers, especially when the comparison spectra are more separated. This problem has been theoretically developed by Ditscheiner.

Sometimes the slits are made somewhat curvilinear, then the lines obtained are rectilinear, at least for a certain portion of the spectrum.

The spectrocomparator, invented by Hartmann, serves for differential measurements. The value of this instrument lies not so much in the fact that it enables us to obtain a rapid determination of the difference of velocities upon two spectrograms of the same star or of different stars as in the new principle introduced into astrospectroscopy. This new principle permits us in the determination of radial velocities of stars to curcumvent to a considerable extent the complex problem of the system of wave-lengths.

In the spectrocomparator (fig. 24 and 25) two spectrograms may be investigated in the field of a double articulate microscope. The images, obtained after their passage through the properly directed objectives O_1 and O_2, of the same spectral portion are reflected by two rectangular prisms P_1 and P_2 against one another and they get into a special optical system (Prismenkorper) P_3 P_4 which consists of two rectangular prisms of unequal size glued to each other. The hypothenuse of the second, smaller prism is so silvered that at its center appear three separated narrow little bands. Thus the rays coming out from one objective proceed along the hypothenus of the small prism and they will reflect upwards only in those places where the little bands are. We shall see the cut out parts of the spectrum of one of the stars (middle band) and of the comparison spectrum (side bands), if the mutual position of the band is placed in accordance with what takes place on the spectrogram. The rays from the second spectrogram, having passed through the second objective and been reflected from the cathetus side of the large prism, will be able to pass through the non-silvered parts of the second prism, i.e. between the bands. Then, on both sides of the spectrum of the first star will appear the spectrum of the second star, and on both sides of the upper and lower comparison spectrum for the first star will appear the comparison spectrum for the second star. By means of an eye-glass, common to both objectives, the same small portion of the spectrum on both spectrograms may be observed simultaneously.

The small prism may also be differently silvered, in half, for instance. Then, the upper half of the field will be occupied by the first spectrum and the lower by the second. The stellar spectra or the comparison spectra may be brought to touch each other by the rotation of the screw Q.

While describing the method of reduction of spectrograms it was shown how it is possible, by means of computations and graphic constructions to compare two spectrograms which differ somewhat in dispersion, i.e. how to reduce them to a normal dispersion; the latter, as is known, enables us to shorten the work on the spectrograms. In the spectrocomparator both spectrograms are made comparable because of a simple, but remarkable optical method which consists in the following: moving by the screw V, the general part of the microscope containing the eye-glass and the optical system glued together, we lengthen one microscope and shorten the other. In this way we change the magnification of each of the microscopes. Having selected the proper position of screw V_1 we can obtain over a small portion of the spectrum complete coincidence of the lines of the comparison spectra of both spectrograms, although in reality their dispersion is different. If the radial velocities are different, the stellar lines in both spectrograms will be shifted with respect to each other.

The measurements by means of the spectrocomparator are simple and fast.

The standard and the investigated spectrograms are both put into one frame under the objective, they are focused and oriented in the usual way by means of screws D_1 and D_2 (see figure 25, page 148 in book). The handle H serves to draw the microscope closer to or remove it from the frame. This often facilitates the general focusing at the given magnification. By screw Q the entire microscope is moved until the basic or standard spectrogram is situated symmetrically with respect to the center of the field. The same thing is also obtained for the spectrogram under investigation by means of screw G. Then the two spectra will be seen on the common field in their respective places (see figure 26, page 149). Next, by rotating screw V, the line of iron is made to coincide. But in changing the magnification of the microscopes we spoil the focusing which may be corrected each time by moving the objectives. A few such consecutive operations are sufficient in order to obtain, in good focus a combination of lines of the comparison spectra in a given portion. Having made on the head of the micrometer screw S the reading corresponding to the coincidence of the lines of the comparison spectra, the head is rotated again until the stellar lines on both spectrograms coincide. This difference may be translated into kilometers by means of a table as was explained previously. Such comparisons are made for a large number of portions of the spectrum by moving the frame consecutively with handle K. All measurements are made in the usual way by changing the positions of the spectrograms by 180°. There must be good and uniform illuminations. The basic or standard spectrogram of the Sun or some star must previously be carefully measured on the usual measuring instrument. Since all measurements are made within several divisions of the head of the screw, it is desirable, in the rotation of the plate by 180°, to make readings at such a position of the head as differs by half a revolution with respect to the first position, so that the periodic error of the screw may be eliminated.

Translating all differences of the readings into kilometers, it is possible to take the mean in the usual manner. But, since the dispersion decreases toward the red portion and the accuracy thus diminishes, it is desirable, in the formation of the mean, to introduce weights (for separate portion of the spectrum)

which are inversely proportional to the translatory factor S (the number of kilometers corresponding to a single revolution increases with the wave-length). Let the shift, measured in revolutions, for the given portion at the two positions of the spectrogram be d_1 and d_2, and the mean value be $d = \frac{d_1 + d_2}{2}$. In kilometers it will be dS with weight $p = \frac{1}{S}$ The mean for a large number of portions will be

$$\frac{\Sigma p d S}{\Sigma p} = \frac{\Sigma d}{\Sigma \frac{1}{S}} = \frac{\Sigma d_1 + \Sigma d_2}{2 \Sigma \frac{1}{S}}. \qquad (32)$$

Thus, if we compute before hand, once for all, the table $2 \Sigma \frac{1}{S}$ for various limits of portions covered by the measurements, the finding of the mean velocity is a simple matter, since it is not necessary to translate each time d_1 and d_2 into kilometers. If we add to the velocity thus obtained the velocity of the basic or standard star, we shall then obtain the velocity sought.

The measured radial velocities should be reduced to the Sun, i.e. they should be free of the influence of motions of the earth around its axis and around the Sun. As to the second correction, it is computed by the formula:

$$v_\varepsilon = -V_\varepsilon \sin(\lambda - \odot + i) \cos \beta. \qquad (33)$$

This formula gives the projection Va of the velocity of motion of the earth around the Sun (V_a directed to the point of the ecliptic $\odot + 270° - i$) on the line connecting the observer with the star (of longitude λ and of latitude β) The quantities V_a and i are given with the solar longitude, $\odot$ as argument. In formula (33) 90 - i is the angle which the radius - vector forms with the tangent of the terrestrial orbit at the given point. The Correction V_a may change within the limits of + 30.3 to - 30.3 km.

The correction for the rotational motion of the earth is computed by the formula

$$V_\alpha = -0.47 \sin t \cos \delta \cos \varphi. \qquad (34)$$

This formula gives the projection 0.47 $\cos \varphi$ of the velocity of the motion of the observer (because of the daily rotation the observer approaches the eastern point of the horizon) on the line connecting the observer with the star (of declination δ and hour angle t). Correction Vd, which never exceeds 0.4 km, is negative when the star is west of the meridian and positive when the star is east of the meridian.

12. Systematic and Accidental Errors in the Determination of Radial Velocities. The stellar radial velocities determined with the aid of the spectrograph contian inevitably numerous systematic and accidental errors, and the distinction between the latter is often impossible. Small systematic

errors which cannot be explicitely separated will enter the result as accidental errors. Basically, the errors of radial velocities are of two kinds: 1) errors due to the instruments used and 2) errors connected with the system of wavelengths (15). Besides, some errors are due to outside factors such as, for instance, atmospheric dispersion.

One of the errors of the first kind is that which is connected with uneven illumination of the objective of the collimator. When the illumination is not uniform a certain loss of light will occur and it will cause numerous systematic errors as we shall subsequently see. We shall illustrate this by taking a somewhat artificial case when only half of the objective of the collimator is illuminated by a star. Correspondingly only one half of the objective of the camera will be illuminated after the passage of the light through the prism. Let us denote by d the distance of the center of the collimator from the center of the illuminated pencil of rays. If the length of the camera be f and the error of the focus be Δf (the error may be caused by the inaccuracies of the setting of the focus, of the temperature effect, or by the flexure, or of the bending of the plate in the plate holder), it may be easily seen from figure 27, that the spectral line corresponding to a certain monochromatic pencil of light will occupy position A_1 instead of the normal position A. The shift of line AA_1 or ΔA is equal to:

$$\Delta A = \frac{\Delta f}{f} d. \qquad (35)$$

If the objective of the collimator is illuminated uniformly, a small deviation from the focus courses a wash-out of the spectral lines, but the positions of the lines remain unchanged. Since the objective of the collimator is usually illuminated uniformly by the source of light producing the comparison spectrum, the position of the lines of the latter remain normal. In astronomical practice various degrees of uneven illumination of the objective of the collimator by a star happen quite often, and thus a systematic error is introduced. The extent of such an error can be seen from the following example: if we apply the conditions of figure 27 to the Simeis spectrograph of one prism, then, even for Δf = 0.2 mm the systematic error in the determination of radial velocities is about 10Km. It is necessary always to pay much attention to this circumstance, for the source of many systematic and accidental errors in the determination of radial velocities can be traced to it. Let us enumerate the basic causes of the uneven illumination of the objective of the collimator: 1) inaccurate centering of the collimator with respect to the telescope; 2) the unsatisfactory centering of the correction lens in visual refractors with respect to the objective of the collimator as well as with respect to the objective of the refractor; 3) astigmatism and spherical aberration in the optics of the telescope; 4) the imperfect centering of the reflector, producing an uneven distribution of brightness in the extra focal image which is due to the eccentric position of the little dark circle in the center caused by the hyperbolic or Newtonian mirror; 5) the unevenessin re-

figure 27

flective power of the silver pelicle on the mirrors because of uneven thickness and structure.

All these circumstances, taken singly and together, are the cause of accidental as well as systematic errors in the determination of the radial velocities of stars.

Under certain circumstances this effect may be so large that the determination of the radial velocities of stars becomes entirely impossible.

To a smaller extent these causes have an influence on the contours of the spectral lines in spectrophotometric investigations.

It may be seen from the formula (35) that with uneven illumination of the objective of the collimator a distortion of radial velocities occurs when the focus is not entirely accurate. However, it is possible that the focus is accurate and yet the radial velocities are distorted. This happens with uneven illumination of the objective of the collimator due to the heterogeneous structure of the prism and to other deficiencies in the prism. The latter produce an abnormal refraction. We may assume that the entire prism, or part of it, acts as a lens. As a result the radial velocities may be greatly distorted, as the objective of the collimator is always uniformly illuminated by the source of light producing the comparison spectrum. (The latter is easily obtained). Besides, the spectrum loses in purity with such a prism.

Therefore, before we start a long series of observations it is necessary to investigate the homogeneity of the given prism (or system of prisms) and the extent of distortion of the radial velocities if the prism is not entirely homogeneous.

We have spoken previously of the laboratory method of investigation of the homogeneity of prisms. Now we shall describe a practical method by which this effect may be studied. Having closed by a semicircular diaphragm the lower half of the objective of the collimator, the spectrum of the sky is photographed, and the comparison spectrum is obtained with full aperture of the same objective. Next, on a second plate the spectrum of the sky is photographed, having closed the upper half of the objective of the collimator, and again the comparison spectrum with full aperture is obtained alongside. Comparing the relative shift of the lines in the sky spectrum on these spectrograms, we may get a conception of the extent of the effect of the heterogeneous prism on the determination of radial velocities for uneven illumination of the objective of the collimator. Likewise, it is possible to judge of the effect of heterogeneity of the prism in another direction by closing alternately the right-hand and left-hand half of the objective of the collimator. It is necessary at first to make sure that the focus is accurate and then to take a series of photographs on both sides of the focus in order to exclude the effect of the focus. If the diaphragm on the slit permits, it is desirable to obtain side by side two spectra of the Sun, one having passed through the upper half of the collimator, the other through the lower half.

In order to be albe to judge of the effect of an erroneous focus with uneven illumination of the objective of the collimator, we may do as follows: one plate, at the normal focus, we photograph the spectrum of the sky, having covered one of the halves of the objective of the collimator and obtaining the comparison spectrum with full aperture. On the other plate we obtain a similar photograph after shifting the focus by about 0.5mm. At the same positions of the focus we may obtain a photograph of the spectrum with the full aperture of the objective of the collimator. All of these photographs enables us to judge of the

magnitude of the error that we are interested in for a shift of focus of 0.5mm. It is yet necessary to add here the photograph with the objective of the collimator illuminated by a star (see above) in order to get a conception on the true illumination of this objective. Taking the latter into consideration it is possible to find the approximate value of the error of the radial velocity of a star for the usual error in the setting of the focus of 0.1mm.

However, we must say that even with uniform illumination of the objective of the collimator and with an excellent prism a noticeable systematic error in the determination of radial velocities may remain, if there be any fault in the focus. This may be due to the following two causes: 1) the inclination of the plate holder and 2) the difference in the absorption of the prism between its upper part and its base. As was previously said, a large inclination of the plate holder to obtain a considerable portion of the spectrum is required for certain systems of objectives used in cameras, especially for the Zeiss "Chromat". This inclination reaches sometimes 20° and more. For such an inclination the deviation from the focus leads to a somewhat asymmetric form of the spectral lines in the stellar spectrum as well as in the comparison spectrum. However, this effect will be different for the absorption lines of the stellar spectrum and for the bright lines of the comparison spectrum, and as a result we shall have a systematic error in the determination of radial velocities. For instance, on the one-prism Limeis spectrograph where the inclination of the plate holder reaches 22°, a deviation from the focus of 0.2mm leads to an error of 2.4 km. The effect of the uneven transparency of the prism also enters here. For this reason it is necessary to avoid such camera objectives which require a large inclination of the plate holder. This circumstance is a great deficiency in spectrophotometric investigations of contours of lines.

As was indicated in §2 the absorption in a prism considerably increases toward the base, and, hence, if the objective of the collimator is uniformly illuminated, the objective of the camera in the given monochromatic light will be illuminated unevenly, and if there exists a certain deviation from the focus, then the contour of the spectral line will be somewhat asymmetric even when the plate holder is perpendicular to the axis. As in the previous case, the character of this asymmetry will be different for lines of the stellar spectrum and of the comparison spectrum, and as a result a systematic error may occur in the determination of radial velocities as well as in the study of contours of spectral lines.

The previously mentioned errors connected with the uneven illumination of the objective of the collimator, with the inclination of the plate holder, with the deviation from the focus, and with other causes (which are not often easy to distinguish one from the other) increase as is easily seen, with the decrease of the focal length of the camera, since the same linear shift is connected with the increase of shift in kilometers as the scale diminishes. This is the reason why it is necessary, in photographing the stellar spectra by means of cameras of short focus, to investigate carefully all the above-mentioned effects which may distort the value of radial velocities.

These are mostly systematic errors; however, accidental errors are also possible. Depending on the focusing of a star on the slit, on the quality of the image, on the method of guiding, and on other causes, it may happen the uneven illumination of the objective of the collimator will change from star to star, from one exposure to another. We must also keep in mind that the systematic error of this kind may change depending on the declination and the hour angle because of flexure in the telescope, because of general flexure in the spectrograph, and because of atmospheric dispersion.

In contemporary spectrographs of systematic errors of this nature are mostly small, but the uneven illumination of the objective of the collimator together with the flexure (especially in times past) were the chief sources of errors which sometimes precluded the determination of radial velocities of stars.

Somewhat different in character is the systematic error connected with the imperfect optics of the spectrograph. We shall speak now a few words on the so-called coma-effect. It is known that wide-angled objectives having a considerable field far away from the optical center produce a "coma" or a comet-like appendage the size of which increases with the distance from the center (see the Introduction to this course, §18). If we deal with the spectrum, we shall have the appendage for lines far away from the center of the spectrum, in the case of the "coma effect", directed away from the center. Although the determination of radial velocities of stars is of a strictly differential character, and the lines in the spectra of stars are "attached" to the lines of the comparison spectrum, the coma will nevertheless influence differently the position of the bright lines in the comparison spectra and of the absorption lines in the stellar spectrum. Depending on the position of the line in the spectrum or on its wave-length, a certain systematic error originates. When such an error, which depends on the wave length, appears, its source may be found. As the "coma effect", evidently, will be the more noticeable the longer the comparison spectrum is exposed, two comparison spectra are obtained on the same plate for its investigation; one is somewhat overexposed, the other is somewhat under exposed. If the relative measures show a systematic dependence on the wave-length, then, evidently, we have to do here with the "coma-effect". The dependence of the systematic error on the wave length may also be due to the inclination of the plate holder which is not difficult to detect.

The systematic and accidental errors are closely connected with the width of the slit. As was indicated in 5 the widening of the slit is a simple and excellent device to increase the light gathering power of the spectrograph. However, we must take into consideration here the posibility of decrease of the accuracy in the determination of the radial velocities of stars with respect to the increase of the accidental and systematic errors. The decrease of the accuracy in the determination of the radial velocities are due to the following causes: 1) the decrease in the purity of the spectrum P = $\frac{\lambda}{\varphi\alpha+\lambda}R$ [which is inversly proportional to the width of the slit, see formula (4) of §1] determines the difficulty in identifying the spectral lines and especially in the increase of the number of "blends" i.e. of lines superimposed one upon the other, 2) because of the widening of the lines and the decrease of their sharpness, brightness, and contrast, the error of measurement will increase, 3) with a wider slit the position of a star may not be entirely symmetrical with respect to its edges, and this fact causes a systematic error, especially when the exposure is of short duration. In reality, the measurements of the spectrograms, obtained with slits of various width, do not show any noticeable increase of the possible error of setting on the lines by means of the increase of the width of the slit from 0.025 to 0.060, whereas the gain in exposure is about twice as much. However, the probable error of the spectrogram, in general, increases with a wider slit especially when the exposure is of short duration. Assuming that a star on the slit is 0.01mm. above or below the middle (which is quite possible with a width of 0.060mm of the slit, with good images, and with a short exposure), we shall obtain a shift of spectral lines equal to 0.01 f/f_1, where f and f_1 are the focal lengths of the objectives of the camera and of the collimator. In long cameras f/f_1 is about 0.75, and hence, with the usual dispersion of the order 36A = 1mm near Hγ the velocity

of the star will be distorted by a quantity of the order of 18km. Even with a width of 0.05 mm of the slit the star may be held closer to one edge than to the other. In this connection much attention should be given to the edges of the slit which should be equally sharp. A slight difference in the form of the two edges may lead to a systematic shift towards one edge thus leading to a systematic error in radial velocities. The figure of the stellar image, which in case of astigmatism of the mirror or objective will be not quite circular, is also of importance here.

Sometimes there appears a systematic difference in radial velocities of the same star depending on whether the spectrogram is obtained before or after passing the meridian. This difference is of the order of 0.8km for the spectrograph at the Lick observatory. If there exists a bending of the tube, the objective of the collimator will be differently illuminated in the eastern and in the western position of the telescope as well as at large and at small zenith distances, since the image of the illuminated objective or mirror of the telescope is projected upon the objective of the collimator. To be sure, the effect of bending in the telescope is difficult to differentiate from the effect of general bending in the spectrograph. Likewise, this effect at large zenith distances is difficult to separate from the effect caused by the atmospheric dispersion.

In instruments of long focus, even at zenith distances of 45°, the image of a star has an elongated form which may lead to an asymmetric position of the star on the slit. In connection with atmospheric dispersion there may arise systematic and accidental errors of the order of several kilometers. Usually we avoid photographing the spectra of stars for the determination of radial velocities when the zenith distance exceeds 50° - 60°.

We may emphasize that errors of uneven illumination of the objective of the collimator and of asymmetric position of a star on the slit increase as the focal length of the camera and the linear dispersion decrease.

A small error in the determination of radial velocities of stars may be made because of the so-called pole-effect. This is connected with the following circumstance: during the burning of the voltaic-arc the wave-lengths of the comparison spectrum change somewhat when the luminous pair are closer to one of the poles.

The accidental error in the determination of the velocity depends on the number of measured lines and on their sharpness or diffuseness.

It is also necessary to keep in mind that a large error of the mean velocity, based on the measurement of four or five spectrograms, is sometimes connected with a real change of the velocity itself (the so-called spectroscopic binaries of small amplitude, see part III chapter 2).

The systematic error connected with the personal equation of the observer deserves our attention. For a series of measurements made by several observers this error reaches 0.5-1.0km.

Finally, there are many cases in astronomical practice when a temporary systematic error arose due to unknown causes. Such an error arose, for instance, at the Lick observatory in spite of its excellent equipment and well trained observers.

Sometimes a systematic error changes with the years, but it is not always possible to trace its cause.

Other systematic errors determined by radial velocities will be investigated in §§20 and 21.

§13. The Effect of Temperature. The Thermostat. One of the most serious sources of errors in astrospectroscopy, especially in the determination of radial velocities is the temperature effect. To it are subject the refraction of rays in a prism, the focal lengths of the lenses of the camera and collimator, the length of the metallic tubes of the camera and collimator, and, finally the length of all other metallic parts of the spectrograph. These changes, together or separately, may cause large accidental errors if no measures are taken to make the temperature effect as small as possible. The early determinations of radial velocities gave errors of the order of ten and more kilometers because no measures were taken to protect the optical and mechanical instruments from changes of temperature.

Simply theoretical considerations show that in order to obtain reliable radial velocities it is necessary to creat a constancy of temperature in the spectrograph within the limits of 0°.1. The constancy of temperature needs be also maintained in photometric investigations of contours of thin lines, since a change of only a few tenths of a degree causes a certain blur in the lines. In the latter case the temperature changes are even more dangerous, since in the determination of radial velocities, because of the exposed comparison spectrum (at the beginning and the end of the exposure, at least), we have essentially to do with the differential temperature effect, whereas in the photometry of the lines the effect enters fully.

First of all it is desirable that the daily temperature fluctuations in the dome be as small as possible. This depends on the selection of a place and its character, and on the structure of the dome and its thermic properties. It is further necessary that at the beginning of the observations the temperature inside the spectrograph be as close as possible to the temperature of the outside air. The most important thing, however, is the construction of a good thermostat and the proper functioning of the latter. The thermostat is a large, wooden (or of other material) box which includes almost the entire spectrograph, leaving outside only the very end of the collimator with the slit and the contrivance for guiding. The thermostat serves for the purpose of maintaining of a constant temperature in the spectrograph. In order to decrease the thermal conduction it is desirable to line its inside walls with felt, and to place the entire thermostat in a sack. Inside the thermostat, along its walls, a series of wires are fastened on insulators so that the electric current of the usual electric lighting system may go through the wires. This system of wires should go through many parts of the spectrograph in order to maintain a more uniform temperature inside the latter. However, the wires should not come close to the optical and metallic parts of the spectrograph in order to avoid direct radiation. Two or three ventilators in the spectrographs are desirable for a more uniform distribution of temperature. Next, it is necessary that the separate metallic parts of the spectrograph of the system supporting it (which are subject to the action of the outer air) be covered with felt or other non-conductor, as otherwise harmful effects may take place with temperature changes.

If we let the electric current pass through this system of thin wires (made of a material of great resistance such as manganese and others), heat will be generated inside the spectrograph which will compensate the coolness on account of the natural fall of temperature in the dome. The constancy of temperature is regulated automatically by means of one or two mercury thermometers situated

usually near the most important parts of the spectrograph. The mercury regulator thermometer must be very sensitive (having a large bulb contained in thin walls and the mercury capillary should be very narrow). In the upper open part of this thermometer a thin platinum wire is movable and a platinum contact is soldered to the lower part. It has a connection with the mercury in the bulb. The entire thermometer is a part of a chain of low voltage which automatically closes and opens with a small change of temperature. The action of the thermostat is schematically represented in figure 28.

The heating action of the thermostat depends on the force of the current and on the resistance of the wire $(\sim I^2 W)$. Hence the resistance of the wire W may be computed if we know the mean force of the working current I. Since the resistance must be equal to 100 ohms in order to obtain 400 watts with a current of two ampers. Depending on the difference of the temperatures in the spectrograph and the dome and depending on the rate of fall of night temperature, it is necessary to vary the heating action of the thermostat. This is done in the following way: a rheostat is inserted into the heating chain. This rheostat consists of a series of lamps connected in parallel by means of which the force of the current may be regulated. If, at the beginning of the observations, we close the low-voltage chain, including the relay by moving the little platinum wire in the mercury thermometer-regulator, the amature is thereby attracted to the electromagnet. Thereby a break in the high-voltage heating chain is made, since the heating circuit goes through the armature. Next, with the constant lowering of the temperature, the level of the mercury column will tend to lower and at a certain moment the mercury will go down from the end of the little platinum wire; a break again takes place in the heating chain, etc. During the closing and breaking of the high-voltage chain the lamps go out and get lighted again and they thus produce signals telling us as to the action of the thermostat. The mercury thermometer-regulator should be so sensitive that, as the temperature lowers the heating should begin in a few minutes. Otherwise the inertia will be great and the accuracy of the thermostat in maintaining a constant temperature will be lowered. Besides, a direct heating of closely placed parts is dangerous during continuous action.

In order to be able to judge on the influence of the temperature changes on the radial velocities, it is possible to obtain the spectrum of the arc at a normal temperature, and then, (without touching the plate holder) having connected the thermostat and having moved up the little platinum wire, the temperature is raised by a few degrees. Next, the low-voltage chain is connected again, and the thermostat is allowed to work automatically for more than an hour, so that we may be sure that the temperature in the thermostat becomes quite even. After this a second photograph of the voltaic arc is taken on the same plate. This photograph is so taken that, having shifted the diaphragm on the slit, the second photograph appears alongside the first. The same operation in reversed order may be repeated on the second plate, having taken one spectrum at high temperature and the other at a lower temperature.

Having measured the relative shift of the lines and reducing them to the same degree of temperature, we shall obtain the number that is of interest to us. Thus, for the Potsdam 3-prism spectrograph the shift for 1° is equal to 17 km, and for the Simeis 1-prism spectrograph it is 19 km. Keeping the temperature within 0°.15 we assume the errors of 2.5 and 3.00 km respectively. To be sure, these numbers characterize the maximum temperature effect by photographing the spectrum of the voltaic arc at the beginning, middle and end of the exposure, we shall obtain the radial velocity with an error of only a fraction of the values given above. In general, we may expect that the temperature effect increases with the decrease of the dispersion for a given number of prisms, since the same angular shift for a smaller dispersion will correspond to a larger number in kilometers.

Speaking of the temperature effect, it is necessary to take into consideration the changes of refraction in the prisms, the changes in the focal distance of the objectives of the camera and collimator, and the changes in the length of the tubes of the camera and collimator. As was previously said a change in the focus of the camera is harmful. The metal selected for the tubes has such a coefficient of expansion that the change of the focus of the objective is compensated. Brass is such a material. The change in focus is very slight also when the collimator is made of steel and the camera is made of brass. (Victoria).

Cases are known in astronomical practice where harmful thermal conditions were localized in separate parts of the spectrograph or of its supporting parts, and as temperature changes were taking place this circumstance created a tension or a shift in the collimator or camera (for instance, an increase in heat due to the observer). This effect may be verified by artificially heating and cooling the suspected places.

Finally, the temperature effect has this important consequence that as the temperature changes the dispersion changes also. For instance, for the Simeis 1-prism spectrograph we have the following dependence between the two mutual distances of the following two lines:

	t = 0°	t = 24°
4076-4494	12.282mm	12, 334mm.

It is necessary to keep this in mind in developing the spectrograms while reducing the positions of the stellar lines and of lines of the artificial source of the spectrum to a definite dispersion. This is possible to do by means of corrections obtained from the comparison of the positions of lines of the voltaic arc on a given spectrogram with the corresponding positions of the same lines on spectrograms having a standard dispersion by which "normal" were obtained (§11).

4. The Prismatic Camera and the Slitless Spectrograph.

§14. Prismatic Cameras. It was shown in §2 that in the process of using the usual slit spectrograph there is utilized a quite insignificant part of the stellar light, only about 2% in the case of a 3-prism spectrograph. Besides, in this case it is possible to photograph the spectrum of only one star at a time. For this reason a photographic camera with the prism in front of the objective (the prismatic camera) is widely applied for widening the field of spectral investigations. For the sake of comparison, let us say that, by means of a 1-prism spectrograph in combination with the 40-inch reflector at Simeis, it is possible to obtain the spectrum of a star of $7^m.0$. (the dispersion of about 72Å = 1mm near Hγ) with an image of average quality in 70 minutes, whereas a 6-inch prismatic camera of the same dispersion produces the spectrum of a star 1.3 times as fast. The other important advantage is that by the aid of a prismatic camera it is possible to obtain on the same plate the spectra of numerous stars in a region encompassing several degrees.

However, for the same dispersion the purity of spectrum obtained by means of the usual spectrograph is much higher. This is explained chiefly by the fact that in the case of a prismatic camera the source is directly from the star; the quality of the image and the twinkling of the star are shown in the purity of the spectrum. On the contrary, in the slit spectrograph the source of light is the illuminated slit of the collimator, and thus the purity of the spectrum does not depend on atmospheric fluctuations. However, in the case of the slit spectrograph the exposure depends greatly on the qualities of the image: as we previously said the exposure for the same star may change within the limits 1:10 for large instruments. The brightness of the spectra obtained by means of the prismatic camera does not depend on the quality of the image. Unfortunately, it is almost impossible to realize for the prismatic camera the conditions of constant temperature, which fact lowers the purity of the spectrum with long exposures. Finally, it is necessary to remember that the purity of the spectrum depends also on the homogeneity of the prism and the quality of its surfaces, and the latter usually gets worse as the size of the prism increases. Although the large size of a prism helps to increase the resolving power, this fact is in part compensated by the decrease of the factor [§1, formula (2)] for large prisms are usually made of crown glass.

It is possible by a proper inclination of the plate to obtain a large portions of the spectrum are situated on a curve or a straight line which is somewhat inclined to the main optical axis. It is evident that in the prismatic camera it is impossible to incline the plate holder, and therefore it is often impossible to obtain as large a portion of the spectrum in accurate focus as is often possible in a slit spectrograph.

All these properties predetermine the different fields of application of the spectrograph and of the prismatic camera.

The parallel pencil of light, falling upon the prism of the camera, is broken up into a series of parallel monochromatic pencils. Next, the objective of the camera gives the corresponding image for each monochromatic pencil. The linear dispersion here, as in the case of the slit spectrograph is equal to $\frac{d\theta}{d\lambda} \cdot \frac{F}{206\,000}$ where $\frac{d\theta}{d\lambda}$ is the angular dispersion (§1) and F is the focal length of the objective. As F is usually quite large in prismatic cameras (much larger than in cameras of a slit spectrograph), we may have a prism even of a small refracting angle in order to obtain the given linear dispersion. The small refracting angle is advantageous from two points of view: 1) the absorption decreases with the thickness of the prism, 2) a smaller deviation from the optical axis takes place at a smaller refracting angle.

In order to obtain a given star on the main optical axis, it is necessary to incline the entire camera at an angle which ir determined in the case of minimum deviation from the formulae:

$$\left.\begin{aligned} \sin q &= n' \sin p \\ r &= 2(q - p) \\ n' \sin p &= \sin \tfrac{1}{2}(r + 2p), \end{aligned}\right\} \tag{36}$$

where q is the angle of incidence, 2p is the refracting angle of the prism, h' is the index of refraction, and r is the angle of deviation. For a large p the tube must be set on a star the declination of which is different by 20° to 30° from the star under investigation. This leads to poorer guiding because of the difference in the refraction between the former and the latter star.

Using formulae (36) it is easy to compute the angular and linear dispersion when we know p and the index of refraction for various rays.

In the prismatic camera the prism is also set for the angle of minimum deviation according to usual rules; however, it is possible to do this only for stars which are close to the optical center.

It is necessary to widen the spectrum during photographing. This is most frequently obtained by the accelleration or retardation of the motion of the clock-mechanism. However, if the worn gear of the clock-mechanism has a periodic error, bands along the spectrum are unavoidable, and an irregular blackening of the spectrum latterly takes place, especially when the exposure is not very long. A more uniform blackening of the plate is obtained if there exists a device for the micrometric shift of the plate holder or the entire tube (by several seconds of arc in each 5-10 minutes of time depending on the scale).

Let us state the basic formulae for the prismatic camera. Let us imagine on the plate, obtained with aid of the prismatic camera, a system of coordinates whose origin is at the center of the plate and so placed that y-axis is parallel to the refracting angle of the prism, and the positive x-axis goes into the direction of increasing wave-lengths. Next, let us have an ordinary photograph of the same region without the prism, and let the optical axis coincide with the unrefracted normal ray (the normal ray is a ray of that direction and of that wave-length for which the prism is set at an angle of minimum deviation; we distinguish between an unrefracted ray before it enters the prism and a refracted ray after it comes out of the prism). Let us denote the coordinates on this plate by ξ and η in the same way as the coordinates on the plate with the spectra are denoted by x and y.

If the dispersion is infinitely small, $x = \xi$ and $y = \eta$. Using the constants of the prism, it is possible to derive the interrelation between these coordinates (a simple but long derivation was made by Schwarzschild):

$$\left.\begin{aligned} x &= \xi - [P\xi^2 + Q\eta^2][1 - P\xi + A\xi^2 + B\eta^2] + M[1 - P\xi + C\xi^2 + D\eta^2] + M^2E\xi \\ y &= \eta + \xi\eta(x - \xi) = \eta + \xi\eta(M - P\xi^2 - Q\eta^2). \end{aligned}\right\} \quad (37)$$

The constants Q, P, A, B, C, D, E are connected only with the angle of refraction of the prism and the index of refraction, n, for a definite wave-length (the normal wave-length)

$$\left.\begin{aligned} &Q = \left(1 - \frac{1}{n_0^2}\right) tg\, q;\ P = \frac{Q}{\cos^2 p};\ R = \frac{P}{4} - \frac{1}{2} cotg\, 2q; \\ &A = P^2 + RP + \sec^2 p; \\ &B = RQ;\ C = 1 + RP + A;\ D = 1 + 2RQ; \\ &E = 1 + \sec^2 p + 2RP \end{aligned}\right\} \quad (38)$$

M is the only constant which changes for various wave-lengths:

$$M = K + RK^2;\ K = \frac{n_0 - n}{n_0} tg\, q. \quad (39)$$

The constant M may be so interpreted that for $\xi = \eta = 0$ (for a star on the optical axis) $M = x$, i.e. this is the length of the spectrum from a wave-length, corresponding to the index of refraction n_0 to some other given wave-length. It is also said that M denotes "the axis of dispersion".

Although in the derivation of the formulae (37) the terms of the higher order were neglected, these formulae are sufficiently accurate.

It is necessary to keep in mind that the dispersion changes according to the position of the star on the plate. If M is the dispersion on the main optical axis, the dispersion at the point ξ, η is equal to:

$$M(1 - P\xi + C\xi^2 + D\eta^2). \quad (40)$$

Away from the main optical axis the spectra have an inclination with respect to x-axis.

$$i = \xi\eta. \quad (41)$$

If we neglect the dispersion M in the formulae (37), then the quantities

$$(P\xi^2+Q\eta^2)(1-P\xi+A\xi^2+B\eta^2) \text{ and } \xi\eta(P\xi^2+Q\eta^2) \qquad (42)$$

may be interpreted as a relative shift of the images of the spectrum and the star. In practical applications we also must take into consideration the corrections connected with the errors of setting the prism and the camera. The formulae, given above, are applied in the general form in the solution of certain problems connected with accurate determinations of positions of the spectral lines on the plate.

The applicability of the prismatic camera to the photographing of the spectra of faint stars cased several attempts to be made to use it for the determination of radial velocities, for it would have greatly increased our knowledge of the motions of the faint stars (the idea was first proposed by E. Pickering). It is necessary to keep in mind that for many purposes it is important to know at least the mean values of the radial velocities for groups of stars if not the individual values of each star. It is possible in many cases to accept values with a larger probable error if we can obtain mean values for statistical purposes.

Although the attempts of the application of the prismatic camera for the determination of radial velocities were not, on the whole, successful, nevertheless this method may be of great importance in the future. The application of the prismatic camera to the determination of radial velocities may develop chiefly in three directions: 1) the method of an artificial absorbing medium enabling us to obtain the absorption spectrum of this medium in the spectrum of stars; 2) the reversion method based on the principle of relative shift of two spectra of the same star by turning the tube, prism, or plate holder by 180°; 3) the method based on the change of the distance between two lines in the spectrum as a result of the Doppler shift.

In regard to the artificial absorbing medium, the application of filters filled with a water solution of Neodymium chloride, proposed by the physicist Wood, appears to be full of promise. This filter yields the line 4272.90, which is quite sharp and whose width is about 3A, and also a few less suitable bands around H The influence of temperature on the wave-lengths of this solution is small.

In the use of the reversion method (first proposed by Hale and Wadsworth --, There are several variations), a region of the sky is exposed on the same plate, or on different plates, and in the second photograph the spectrum is turned by 180° (one of the methods mentioned above). Both spectra are placed side by side, the mutual distances of the same spectral lines for a series of stars are measured. If for non-moving stars this distance be S, it will be S' for a star moving along the line of sight, and the shift effect will, evidently, be double. If we limit ourselves to stars near the main optical axis, this distance is equal to:

$$S' = \alpha + M(\lambda) + L(\lambda)\, r, \qquad (43)$$

where α is a constant depending on the setting of the instrument, M (λ) is the double distance in the spectrum up to a certain wave-length (or double dispersion), and the term

$$L(\lambda)v = \frac{du}{d\lambda}\cdot\lambda v$$

shows that the shift is proportional to the linear dispersion $\frac{du}{d\lambda}$ to the wave-length λ and to the velocity v. If we take S, as the mean for a large number of stars on the plate,

$$\bar{S} = \alpha + M(\lambda) + L(\lambda)\bar{v} = H(\lambda). \qquad (44)$$

where $\bar{v}$ is the mean radial velocity of the stars on the plate. Then, for an individual star we shall have:

$$S' - H(\lambda) = L\lambda(v - \bar{v}) \qquad (45)$$

This is the equation of the reversion method. In its practical application it is necessary to keep in mind that the stars are often far from the main optical axis, and hence α is not constant.

In general

$$\alpha = A + Bx + Cy - g, \qquad (46)$$

where A, B, C, are constants, x and y are the coordinates on a plate, and g depends on the terms of the second order. Besides, we should take into consideration that the dispersion in the various parts of the plate is not constant, and, therefore, all S' should be multiplied by a factor f in order to reduce them to the dispersion which takes place near the main optical axis.

Instead of (43) we shall have in the general case:

$$S'f + g = A + Bx + Cy + M(\lambda) + L(\lambda)\cdot v \qquad (47)$$

The constants A, B, C may be computed, if the radial velocities of three or more stars on the plate are known. On the basis of the known values of the constants of the prism and of the coordinates on the plate we may compute f and g.

As to the influence of the change of temperature and of refraction, it is usually taken into consideration in the working out of the problem. The reversion method is very rarely used in practice. Schwarzschild successfully applied it to the determination of the radial velocities in the region of the Hyades and of Cygnus. The probable error of the radial velocity for a pair of plates is equal to ± 7. This is evidently the maximum accuracy reached by this method. With the same dispersion the errors in a slit spectrograph will be about half as large.

The reversion method may be applied with a somewhat larger degree of success in the search for stars of variable radial velocity.

Finally, let us investigate the method of determination of radial velocities based on the change of distances between the spectral lines during motion. According to Doppler's principle the shift of the spectral line is equal to

$$d\lambda = \frac{v}{c}\lambda, \qquad (48)$$

where v and c = 3.10^5 km/sec are the velocities of the source and of light respectively. Next, it is known that the interrelation between the wave-length and its position in the spectrum is well represented by the known formula [§ 11, formula (21)] :

$$x_0 - x = \frac{c}{\lambda - \lambda_0}$$

Differentiating (21) and taking into consideration formula (48) we obtain

$$dx = \frac{C\,d\lambda}{(\lambda - \lambda_0)^2} = \frac{vC}{c} \cdot \frac{\lambda}{(\lambda - \lambda_0)^2} \qquad (49)$$

As λ_0 is usually much smaller than λ, the linear shift of dx will decrease with the increase of the wave-length for the same value of v. This also shows that the distance between two lines in the spectrum depends on the radial velocity of a star. It is necessary to keep in mind that this method, as well as the reversion method, should be used only differentially with respect to a group of stars, for the distance between two lines (dispersion) depends greatly on the temperature at which the photograph was taken. In practical application the corrections, mentioned above in the discussion of the reversion method, should be taken into consideration. For the usual values of the radial velocities of stars; the change in the length of the spectrum is very small. This method, proposed in several variants (and first proposed by Orbinsky) has been used still more rarely.

In general we may say that the application of the prismatic camera to the determination of radial velocities was unsuccessful. However, as improvements in the various methods may be developed, we may expect much progress in the future. All attempts were made using a moderate dispersion at which the probable error is about ± 10 km. With a small dispersion, which may be applied in the case of faint stars, we may expect that the error will reach more than a score of kilometers. This fact is the chief obstacle at present in the application of the prismatic camera to the determination of radial velocities of faint stars even for statistical purposes.

We may hope for better success in the application of the prismatic camera to the determination of the radial velocities of some of the so-called anagalactic nebulae of very small angular dimensions and the velocities of which are very large (often exceeding 1000 kilometers). The same statement may be made with respect to stars of very large velocities. The possibility of application is based in these cases on the following considerations; 1) an error of several scores

or even a hundred kilometers may be accepted, for it is only a small fraction of the velocity to be determined; 2) in many cases these nebulae are of very small angular dimensions (in the clusters of nebulae) and differ little in form from stars; 3) in the nebulae clusters the nebulae are so crowded that in the application of the prismatic camera, we shall obtain simultaneously on a single plate, with a long exposure, a large number of spectra (see figure 29). A simple calculation shows that at a velocity of 10,000 km. (which happens frequently) the distance between the lines K and H_γ will decrease by 0.15 mm and more, if the dispersion is so selected that for stationary star the distance between these lines is 2mm.

If at the present time the application of the prismatic camera for the determination of radial velocities is unsuccessful, at least, for stars, the prismatic camera is an excellent device for the spectral classification of stars and for spectrophotometric investigations. In the realm of classification of stellar spectra a great work has been done by means of comparatively modest instruments at the Harvard observatory. The result of this work is the Henry Draper Catalogue containing the spectra of 225300 stars (abbreviated H.D.C.). At present this work is being continued for faint stars (Henry Draper Extension). The Harvard classification (see below) is sufficiently accurate, as may be seen by comparing it with small catalogues obtained with slit spectrographs.

The application of the prismatic camera to photometry of the continuous spectrum is of exceptional importance, as the spectra obtained by means of the slit spectrograph are not suitable for the purpose. This fact is explained by the following considerations: 1) since there are numerous spectra simultaneously obtained on a single plate, the determination with respect to the distribution of energy in the spectra may be made swiftly and accurately; 2) the obtaining simultaneously of spectra of a large number of stars; this cannot be done by a slit spectrograph; 3) the distribution of energy in the spectrum obtained by means of a slit spectrograph may be considerably distorted by atmospheric dispersion, for on the slit we can keep usually only a portion of the spectrum, especially if the Zenith distance exceeds 40°. As to the photometry of spectral lines, the spectra obtained with the prismatic camera are less suitable than the spectra obtained with the slit spectrograph for reasons mentioned above (dependence on the purity of the spectrum, and hence on the contour and depth of lines from the quality of the image, on the guiding, temperature effect, etc.). However, the first successful investigations in this respect were made, to be sure, during the study of spectra obtained by means of the prismatic camera.

§ 15. The Slitless Spectrograph. The application of the prismatic camera to very faint stars meets a serious difficulty in that instruments of large size are required for this purpose. Accordingly, it is necessary to have a prism of a correspondingly large size. The preparation of a large good prism is very difficult, and deformation of its shape is almost inevitable. For this reason another way for the study of the spectra of very faint stars was found. This is done by the slitless spectrograph which is constructed in two ways. In front of the focus of a reflector or refractor is placed a doubly concave or, in general, a dispersing lens the angular aperture of which is $\frac{a}{f}$ and equal to the angular aperture of the objective or mirror of the telescope proper. Having placed the lens so that its focus coincides with the focus of the telescope, we shall find that the converging pencils of the stars in the field of view will become parallel after passing through the scattering lens, which plays here the rôle of a slitless collimator. If the prism be placed immediately in the path of the converging pencil as it obtains in an eye-piece (ocular) spectroscope,

the aberration becomes so large that the purity of the spectrum is very small. According to Rayleigh, the smaller the angle of the cone of the converging pencil, the less is the effect of aberration. The introduction of negative lens in the path of the converging rays enables us to make this angle as small as we wish (it may even be reduced to zero).

However, the lens itself causes some aberration. It can be shown that, with the usual angular aperture for reflectors, only a very small lense of less than 1.5mm in diameter may be used as a collimator; with a larger lens the aberration will be large. Thus, the field in this slit less spectrograph will be very small.

We may overcome this difficulty in two ways. First, we may prepare a positive lens for the objective of the camera, and a negative lens for the collimator, making both of the same material as that for the prism and having their focal lengths equal but of opposite sign. Such a system may be regarded with some approximation as a plane-parallel plate. Had the two lenses been in contact, the total effect of aberration would have been reduced to zero. However, the compensation will be not quite complete, first, because the lenses are separated, and second, because the prism is situated between them (it is assumed here that we deal with a reflector). But the remaining effect will be very small. Such a system of simple lenses enables us to obtain a satisfactory field of the order of 30' for a metric reflector in the ratio of 1:5. The great advantage of this is its great light-gathering power, for the losses from absorption and reflection are very small in simple lenses. Its deficiency consists in the limited focal length of the camera and, correspondingly, in the selection of linear dispersion. A good spectrograph of this kind, made of quartz optical parts has been constructed at the Lick Observatory. Such a system is especially suitable for quartz, for here it is very difficult to correct the aberration in any other way.

However, a slitless spectrograph may be constructed also without the limiting conditions mentioned above. If we take for the collimator an achromatic negative system consisting of crown and flint glasses, we shall be entirely free in the selection of the focal length. It is assumed in this case that the objective of the camera is also achromatic. It is desirable in such a spectrograph to have several cameras by means of which we should be enabled to obtain different linear dispersions and thus to widen the field of application of the spectrograph. It is desirable, however, to obtain a resolving power as great as possible. The latter may be obtained by using two prisms, or at least, one flint prism with a refracting angle of about 60°.

If such a spectrograph is to be used in combination with a refractor, it is necessary, in the estimate of the achromatic system of the collimator to take into consideration the chromatic curve of the objective of the telescope. Using the customary formula for a compound lens we may so select the focal lengths of crown and flint glass depending on the difference of the foci of the two given rays of different wave-lengths (as a consequence of the chromatic curve of objective), that both of these rays will come out of the collimator quite parallel. This will enable us to obtain a large portion of the spectrum in good focus.

In the slitless spectrograph the condition of minimum deviation of rays in passing through the prism is maintained only for the central star. The purity of the spectrum diminishes as the distance from the center of the field increases. Besides, it is necessary to keep in mind that in the slitless spectrograph, when a certain stellar field is photographed, the rays of each separate star encompass

only a part of the prism as they pass through it. This circumstance depends on the negative system from the focus and angular aperture of the telescope. This necessarily lowers the resolving power and the purity of the spectrum decreases. An objective with an excellent field should be selected for the camera. The greater the light-gathering power of a spectrograph, the shorter the camera and the larger the angular aperture, the more difficult is it to realize. Besides, it is evident that the field should be quite perpendicular to the optical axis of the camera.

The slitless spectrograph as well as the usual one, should be contained in a thermostat. Serious attantion should be paid to the mechanical construction of the slitless spectrograph as the problem of bending in very long exposures is of great importance.

The idea of constructing a slitless spectrograph came from Wadsworth and Campbell.

Another variant of the slitless spectrograph may be made only in combination with the reflector. In front of the focus of the main mirror, in the path of the converging pencil, a convex parabolic mirror is placed the focus of which should coincide with the focus of the main mirror. In this case the reflected rays will go back in the form of a parallel pencil. If the main mirror is perforated, we may set directly the prism and the photographic camera. If the mirror is not perforated, the parallel pencil is thrown back, by means of a mirror set at an angle of 45°, to that side where the prismatic camera is fastened. Such a spectrograph, which is not quite as good as that of the first variant, was built at Greenwich.

The slitless spectrographs, because of their great light-gathering power, have a great future in their application to the study of the spectra of faint stars and of other very distant objects.

5. The Spectral Classification

16. The Base of the Harvard Classification. The application of the spectroscope to the study of stars revealed so many characteristic differences in the stellar spectra that a spectral classification became possible even during the stage of the visual method of observation. Such, for instance, is Secchi's classification. The next stage of development was Vogel's classification the bases of which was the principle of continuous passing from one spectrum to another.

A detailed classification of stars was developed about forty years ago by Pickering and Cannon at the Harvard Observatory. The following letters are used for the designations of the types of spectra: O, B, A, F, G, K, M, and for greater detail numbers from zero to 9 are added on the right-hand side to the letters. The original designations of the sequence of spectral types followed the alphabethical order, but subsequently the places of certain letters had to be changed because of newly discovered facts while other letters were dropped out because they were based on erroneously interpreted spectra. When this classification was being developed, very little was known about the factors which determine the differences of stellar spectra. Besides, in the development of the Harvard classification no theoretical considerations whatever were taken into account. Therefore, this remarkable classification is a purely empirical one.

It should be noted that the Harvard classification is based on general evaluations of the relative intensities and form of the spectral lines and not on the distribution of brightness in the continuous spectrum. This assumption is correct from the theoretical and practical points of view. Indeed: 1) the interrelation between the type of spectrum and the distribution of brightness in the continuous spectrum is very complicated. We know of cases in which a star of type B5 shows the same distribution of brightness in the continuous spectrum as a star of F5 type, at least, in the photographic and visual portion of the spectrum; 2) even if the interrelation between the type and the distribution of brightness in the continuous spectrum were very simple, the method of classification based on the distribution of brightness in the continuous spectrum would have still been inaccurate for stars of very high temperature (white stars) and very distorted by the influence of molecular bands in case of stars of very low temperature (red stars); 3) if in the continuous spectrum a deviation from the laws of radiation of a black body occurs, its various regions will not yield identical results; 4) from the practical point of view, the study of the distribution of brightness in the continuous spectrum is a complicated and protracted process. To be sure, a spectral classification based on the distribution of brightness in the continuous spectrum is in itself valuable and interesting, but a classification based on spectral lines has numerous advantages over the former.

The Harvard classification has proven to be an unusually practical one. It has remained unchanged up to the present with the exception of the most extreme regions of the spectral series (near O and M). We are certain that in the future, when new factors will appear and thoroughly understood, a rational astrophysical spectral classification will be developed which will exist side by side with the empirical one and will combine with the latter. But even now the Harvard classification reveals certain facts of great theoretical importance. First of all, the classification established the unity of chemical elements in the stars and on the earth. Next it became clear that more than 99% of the stars fall into the classes B, A, F, G, K, and M, and this consecutive series is linear, strictly continuous, i.e. spectra may be always found which are intermediary in character between the other two regardless of how small the selected interval may be. The classification is also linear in the sense that there is only one way to pass from one class to another. This means that, if we study the intensities of lines of various elements in stars, for instance, of type A0 and G0, we can indicate only one way of passing from A0 to G0 for all these characteristics separately or collectively. This shows that, apparently, only one essential factor (as we shall see later, it is the temperature) determines the changes in the spectrum in passing from one class to another. This fact is of great significance and it also shows that the differences in chemical composition of stellar atmospheres are small; at least, the possible difference in composition has a much smaller influence than the basic factor which is the change in temperature. The latest observations show that a series of other factors (as the electric field in stellar atmospheres, the difference in the gravitational field , the difference in the number of atoms of a given element over a unit of a stellar surface, rotation, etc.) have an influence on the behavior of the spectral lines. Nevertheless, the temperature still remains the dominating factor which determines that rigorous consistency which is revealed by the Harvard classification of stellar spectra. We shall subsequently say more on this matter.

Before we pass on to a detailed exposition of the spectral classification, let us note certain aspects of a general nature for each of the classes (see the table of spectra on figure 30, between the pages 172 and 173 in book)

The spectral classification begins with the stars of type O, the spectrum of which is characterized by lines of oxygen, ionized helium, twice and thrice ionized oxygen, nitrogen, etc. In passing to stars of type B the lines of hydrogen strengthen while all the other lines weaken. In stars B0-B5 the most remarkable lines besides those of hydrogen are the lines of neutral helium and singly ionized oxygen and nitrogen, which subsequently weaken and disappear in A0. In stars of type A hydrogen reaches its maximum intensity and subsequently weakens, although its presence may be traced in the most extreme spectral classes. Alongside with this, near B8-A0 appear lines of ionized metals such as Ca, Mg, Fe, and others. In stars of type F, and sometimes earlier, appear lines of metallic atoms in the neutral state (the so called "arc lines"). In stars of type G and K these lines strengthen, especially the so called "low temperature" lines corresponding to the least degree of excitation of an atom. The lines of ionized calcium reach here their maximum intensity and width, and they remain subsequently almost unchanged. Near K5 appear bands of titanium oxide which rapidly strengthen in stars of type M. In this class the "low temperature" lines strengthen still more and lines corresponding to a stage of higher excitation weaken.

Of late it has become clear that near G5-K there exists a certain branching: 1) the normal basic branch, which is characterized by a developement of titanium oxide lines, passes into M (the overwhelmingly large number of stars); 2) a very exceptional branch of classes R-N characterized by a development of carbon bands; 3) a very exceptional branch of class S the distinguishing characteristic of which are the bands of Zirconium oxide.

It will be shown in part III of this course, in the chapter on ionization (Chapter 3), that these changes in the spectrum are thoroughly explained by the change of the general degree of excitation of atoms, as a result of the fall of temperature along the series.

§ 17. The Spectral Classification of Stars. Let us investigate in detail the Harvard spectral classification. The latter has been changed to large extent in regard to stars of type O. There are two groups of stars of type O; 1) stars of type O the spectrum of which shows exclusively or overwhelmingly lines of absorption; 2) stars of the Wolf-Rayet type the spectrum of which consists of bright (emission) bands. For the first group a classification is introduced which was developed at the Victoria Observatory. The principle of the excitation of atoms, especially those of ionized helium and hydrogen, are the bases of the classification (O0-O9). The attempt to extend this principle to the stars of Wolf-Rayet type leads, so far, to no definite results.

O0 is a hypothetical class for which no representative stars have been found as yet. The temperature is so high that all lines must disappear. It is possible that the nuclei of certain planetary nebulae correspond to O0 (see the chapter on nebulae in Part III)

The subclasses O1-O4 are left for possible future predecessors of O5 .

In the subclass O5 the lines of ionized helium 5412, 4542, 4200, 4026, 3934 3858, 3815 (the so called ζ Puppis lines) and the hydrogen lines are the most prominent ones. The line 4542 is approximately equal in intensity to 0.6 H_γ The lines N^{++}, 4634 and 4642, are noticeable. Representative: B.D. + 35°.3930.

O6. The lines of ionized helium have somewhat weakened the line of neutral helium 4471 is weaker than 4542. The line Si^{+++} 4089 is noticeable, but it is weaker than N^{++} 4097. Representative: B. D. + 44° . 3630.

O7. He 4471 is more intensive than He^+ 4542 which in turn is approximately equal to 0.4 H_γ N^{++} 4097 reaches it maximum intensity. The triplet C^{+++} is noticeable. Representative: 9 Saggittae.

O8. He 4471 is twice as intense as He^+ 4542. The lines of Si^{+++} and of N^{++} are of equal intensity. The triplet C^{++} is strengthened, Mg^+ 4481 appears. Representative: λ Orionis..

O9. Si^{+++} 4089 and 4116 reach their maximum. Si^{++} 4552 and 4568 appear. Mg^+ strengthens. O^{++} 3962 and 5562 are very noticeable. The helium lines strengthen. Representative: 10 Lacertae

The Harvard classification Oa, Ob, Oc, Od, based on the study of bright (emission) bands, is preserved for stars of Wolf-Rayet type.

Oa. The bright band near 4650 is the most outstanding one. H_γ -$H\delta$ are also of an emission character. Other bright bands are also noticeable. Representative: B.D. + 35° . 4013.

Ob. A wide bright band near 4686 is the most characteristic aspect. H_β and H_γ and the lines of ionized helium (series of ζ Puppis) are bright. Representative: B.D. + 35° . 400.

Oc. The bands are narrower than in Oa and Ob. Two bright lines 4686 (He^+) and 4638 (the latter consists of two lines N^+ 4634 and 4642) are noticeable. 4686 is twice as bright as 4638. The lines of hydrogen and of series ζ Puppis are bright. Representative: B.D. + 36° . 3987.

Od. All lines are absorption lines except 4686 and 4638. The lines of absorption of the series ζ Puppis are very noticeable. A very weak line of He , 4471, appears. Representatives: ζ Puppis and λ Cephei.

Such a classification of stars of type O was accepted at the convention of the International Astronomical Union in 1928. A somewhat different classification of stars of type O was developed by Miss C. Payne in 1928 at Harvard.

There is no distict separation between the stars of the Wolf-Rayet group and those of O5-O9. If in the spectrum of a star of the latter group certain lines are observed in "emission", the letter e is added. For instance O8e. If in the stars of the same group a characteristic of the Wolf-Rayet stars is observed, the letters ew or ew! are added (when these characteristics are especially noticeable) for instance, O7ew or O6ew! . If it is desirable to show the existence in the spectrum lines of absorption as well as lines of radiation, the following designations are permissible: γ Velorum O6ew!a.

Let us pass on now to the following classes which are characterized by absorption lines (the intensities are on an arbitrary scale).

B0. The lines of hydrogen strengthen, and the series ζ Puppis weakens. C^{++} 4647 and 4650 are very noticeable. The triplet O^+ 4070, 4072, 4076 strengthens. The lines Si^{+++} 4089, 4116 reach their maximum. The helium lines 4026 and 4471 are of equal intensity. The line 4650 is more noticeable due to the "blend" of the lines O and C. Representative: ε Orionis.

B1. The hydrogen lines may be traced up to H_τ. The series ζ Puppis is hardly noticeable. The lines of helium are more intense and the lines of Si and O are less intensive than in B0. The lines Si^{+++} 4552, 4568, 4574 reach their maximum. Representative: β Canis Majoris.

B2. The lines of helium reach their maximum intensity in B2-B3. The lines of Si^{+++} and O^+ are much fainter. Representative: γ Orionis.

B3. The lines of hydrogen strengthen. The helium lines are at a maximum as in B2. This is especially emphasized by the disappearance, or extreme weakening, of oxygen and silicon lines. The helium lines 3820, 4009, 4026, 4144, 4388, 4471, and 4922 are expecially prominent. Representative: π^4 Orionis

B5. The line k(Ca^+) becomes noticeable. Another line of Ca^+, H is blended with H under small dispersion. The double line S^+ 4128 and 4131 is noticeable, which is intermediate in intensity between the lines He 4026 and 4471. Mg^+ 4481 is approximately equal in intensity to 0.7 H_ε Representative: 9 Tauri

B8. 4026 and 4471 are very noticeable. Mg^+ 4481 is approximately equal to 4471. K is weaker than 4026. Sometimes lines of ionized metals appear. Representative: β Persei.

B9. The spectrum is almost the same as for the following subclass A0 with the exception that line 4026 is noticeable, and line k is weaker than in A0. Representative: λ Aquilae.

A0. The hydrogen lines are at their maximum, or almost at their maximum of intensity. Line K is approximately equal in intensity to 0.1 $H\delta$. The line H(Ca^+) is separated from H_ε if the dispersion is sufficiently great. The line 4481 is the most prominent after that of hydrogen and K. Metallic lines are often noticeable. The lines 4128 and 4131 are noticeable. Representative: α Canis Majoris.

A2. K is equal in intensity to 0.3-0.5 $H\delta$. The lines Mg^+ 4481, Ca 4227, and Fe^+ 4233 are very noticeable. The lines of helium entirely disappear. Representative: δ Ursae Majoris.

A3. Intensity of line k is approximately equal to 0.8 $H\delta$. The metallic lines are more numerous and intense than in A2. Representative: α Piscium

A5. The line K is more intense than $H\delta$ and is almost equal to H + H . The intensity of the hydrogen lines somewhat diminishes. The line Mg^+ 4481 is not prominent. The lines near 4300 whose origin is due to Fe^+, Ti^+, and Fe are noticeable. Representative: Trianguli.

F0. The hydrogen lines are twice as weak as in A0. K is equal to H + H and it is thrice as intense as $H\delta$. The lines of Fe, Ca, and Ti near 4305-4309, forming the so called G-band, are weak. Representative. δ Geminorum.

F2. The spectrum is the same as in F0. Because of the strengthening of the lines 4305 and 4315 the G-band becomes continuous. Representative: Sagittarii.

F5. The hydrogen lines are twice as intense as in the spectrum of G0. The line Ca 4227 (g) standout prominently among others. Fe 4326 is approximately equal to 0.1 H . The "blend" of lines near 4309 is more intense than the "blend" near 4315. The G-band appears continuous from 4299 to 4315. There are few metallic lines. Representative: α Can. Minoris.

F8. It is intermediate between F5 and G0 as to the strength of the hydrogen lines. Certain metallic lines are somewhat weaker than in G0.

G0. The hydrogen lines are not prominent any more, H_γ is 1 1/2 times as intense as Fe 4326 and approximately 3 times as intense as 4338. The "blend" near 4077, $H\delta$ and 4227 are almost of equal intensity. The wide lines of H and K are the most prominent. The continuous spectrum shows less intensity from H_γ to H_ε Representatives: α Aurigae and the Sun.

G5. The hydrogen lines are weaker than in G0. H_γ is fainter than Fe 4326. The lines Si 3905 and 4103 are at their maximum intensity. The fading toward the violet end is very noticeable. Representative: χ Geminorum.

K0. The hydrogen lines are weaker than in G5. H_γ is equal to half of 4326. The line Ca 4227 (g) is almost thrice as intense as in G0. The wide lines of H and K reach their maximum intensity. The continuous G-band from 4299 to 4315 is more noticeable than the line 4227 (g). Certain portions of the spectrum such as 4077-$H\delta$, 4215-4227, 4470-4525, 4614-4698 are brighter than the neighboring lines. Fading in the continuous spectrum from H_γ to H_ε is very noticeable. Representative: α Bootis.

K2. The spectrum is simular to K5. Certain lines, 4227 for instance, strengthen. A great decrease in brightness of the continuous spectrum toward the short wave-lengths occur. Representative: β Cancri.

K5. The most outstanding lines are H, K, and g. The G-band is no longer continuous. The two "blends" Fe 4383-5 and Fe-Va 4405-08 are very noticeable of which the first is the more intense. The molecular titanium bands are beginning to appear near 4761, H_β 4955, and 5167. Representative: α Tauri.

The Harvard classification is used for stars of type M; in addition the Mount Wilson observatory has developed a classification based exclusively on the evaluations of the intensity of the titanium oxide bands. This classification consists of 10 subdivisions: M0, M1, M2 .... M9. The last three subdivisions are essentially for variable stars.

The Harvard classification of stars of type M(Ma, Mb, Mc, Md) remained unchanged; only instead of Ma, Mb, Mc it is recommended to write M0, M3, M8, and instead of Md it is desirable to use the symbols M0e, M1e, M2e ....M8e.

M0 or Ma. The most intense bands of titanium oxide begin at 4762, 4954, 5168, 5445. The spectrum shows bright intervals at 4556-4586 and 4657-4668. Because of the weakness of the continuous spectrum in the violet part, H and K are almost invisible with a normal exposure. The line Ca 4227(g) is the most prominent. The lines of the G-band are well separated, and the line Fe 4315 is very weak. Representative: α Orionis.

M3 or Mb. The edges of the absorption bands of titanium oxide become quite intense and they appear as bright bands. The line of Ca 4227 is very wide and intense. In place of the G-band are seen only "blends" near 4300 and 4306. Noticeable bright bands are observed near 4556-4586 and 4614-4726. On panchromatic plates are noticeable numerous absorption bands of titanium oxide with edges near 5763, 5816, 5857 and others. Representative: ρ Persei.

M8 or Mc The continuous spectrum weakens, and the edges of the bands become more intense than in M0 and M3. The bands of titanium oxide strengthen. Representative: W. Cygni.

M0e, M1e, M2e .... M8e or Md. The spectra are those of stars of the type M, but, at least, one of the hydrogen lines is an emission line. The spectra of this subgroup differ in details. Most of the long period variable are in this group. Representatives: χ Cygni and O Ceti.

R0. The distribution of intensity in the continuous spectrum is almost the same as in G5 or K0. The H and K lines are clearly visible. The dark carbon band near 4700 is very wide and intense. The dark "blend" near 4395 (probably the origin is due to Ti$^+$, Va$^+$, and Fe) is approximately equal in intensity to the G band. The lines Ca 4227, Fe4234, 4236, and 4239 are quite prominent. There are certain stars which should be regarded, from the character of their spectrum, as intermediate between K and R0. Representative: S.D. - 10° . 5057.

R3. H and K are noticeable but they are weaker than in R0. The continuous spectrum of H toward the short wavelengths is almsot twice as weak as in R0. Representative: B.D. + 5° . 5223.

R5. The continuous spectrum f 4240 toward the short wavelengths is hardly visible on photographs of normal exposure. We may note three regions of noticeable intensity near 4300, 4400, and 4840. The relative brightness is 3, 6, and 10. Representative: S.D. -3° . 1685.

R8. The continuous spectrum is very faint from 4240 toward the shorter wavelengths. Representative: B.D. + 61° . 667.

N0 or Na. The spectrum between 4240 and K is much weaker than in R8. The bright wide bands mentioned in R5 near 4300, 4400, and 4840, are also in N0, (the relative intensities are 0.8 and 10). Representative: 19 Piscium.

N3 or Nb. The relative intensities of the three regions mentioned above are respectively 0.6 and 10. Representative: B.D. + 67° . 350.

S. To this class belong certain long period variables and other red stars which cannot be placed in M or R or N. Absorption bands of zirconium oxide are noticeable near 4650 and 6470. The spectrum between 4500 and 4700 is very complex having numerous absorption and emission lines. Most stars of class S have bright hydrogen lines.

The above classification of stars is based on the study of the portion of the spectrum between 3900 to 5000 A. It is desirable to add to it the characteristics of the ultraviolet, red, and infrared regions.

Figure 30 shows the characteristic spectra of stars of various spectral classes.

§18. A Preliminary Spectral Classification of Nebulae and Novae. The spectral classification of gaseous nebulae and novae stands somewhat apart. There exists the old Harvard classification for the nebulae spectra consisting of a series of bright lines. Besides, Miss Payne has recently attempted to build a rational classification of gaseous nebulae in the light of Bowen's discovery of the identification of nebular lines.

In the old classification the following subclasses exist: Pa, Pb, Pc, Pd, Pe, and Pf.

Pa. The double line 3726, 3739 is more prominent than the main nebular lines 5007 and 4959. Representative: I.C. 418 (Dreyer's Index Catalogue of nebulae).

Pb. The lines 5007 and 4959 are more intense. Representative: The Orion Nebula.

Pc. The line 4363 is the most prominent. Representative: I.C. 4997 .

Pd. The line 5007 is the most prominent. Representative: N.G.C. 6826 (N.G.C. is Dreyer's New General Catalogue).

Pe. It differs from Pd by the appearance of line He$^+$ 4686. Representative: N.G.C. 7662.

Pf. The emission (bright) band near 4650 (probably due to Carbon) is the most characteristic part of the spectrum. Representative: N.G.C. 40.

In Miss Payne's new classification, which is not yet very widespread, new symbols are introduced: P0, P1, P2 .... P10. In the early subdivisions the lines 5007 N$_1$ and 4952 N$_2$ (O^{++}) are relatively very bright. In P8 these lines disappear and the line O$^+$ 3729 as well as those of hydrogen remain. In P9 we have emission lines and a continuous spectrum, and in P 10 we have the continuous spectrum alone (possibly with some absorption lines). The spectral class of the nebula which shines with reflected light is denoted by the inclusion of the spectrum of the star in parentheses, as for instance P10 (B5). The old Harvard classification as well as that of Miss Payne are preliminary classifications.

As to the spectral classification of novae, there exists a preliminary scheme by means of which we may encompass the observed cases. The following subclasses are differentiated:

Qa. An absorption spectrum consisting of weak lines and faint emission bands.

Qb. The absorption lines are stronger, many of them are doubles; the emission lines are more noticeable.

Qc. A spectrum of absorption of atoms ~~of atoms~~ of O, N, He, and of other elements. There are emission lines of the same elements.

Qu. Wide "nebular" bright bands near 3480, 4515, 4640, sometimes they appear also near 4379. Other radiation also appears.

Qx. Bright bands which are due to ionized oxygen, nitrogen, and helium. Weak absorption lines.

Qy. The same as in Qx. In addition there are bright nebular bands.

Qz. Bright nebular lines. Weak bands characteristic of the Wolf-Rayet type of stars.

Intermediate stages are denoted by an additional letter. For instance: Qbc or Qcb, depending on whether Qb or Qc predominates.

The Harvard classification of stars is accepted everywhere. However, observatories, where spectral investigations are carried on by means of slit spectrographs, evaluate the spectra independently, but they do so on the bases of the Harvard classification. For instance Mount Wilson gives a more detailed gradation, so that such denotations as F7, G1, G4, K4, etc. are found there; the Victoria Observatory introduced the line 4227 as a criterion for the subdivision of stars from K5 to M0. Occasionally, systematic deviations obtain between spectral classifications based on the evaluation of spectra obtained with the objective prism and those obtained with the slit spectrograph. For instance, this occurs near A0, as the chief criterion here is the thin K line, the form of which depends on one hand on the resolving power of the spectrograph and its dispersion, and on the other hand, on the exposure of the spectrogram, whereas the given region in the case of a slit spectrograph is often underexposed.

§19. Symbols Denoting the Various Characteristics of Stellar Spectra.

The Harvard Classification indicates the exact positions of stars in the series of O, B, A, F ..... Although a rational physical classification has not yet been developed, but from early times new symbols were added to the old ones, and these new symbols were used to illustrate the form of the lines and even certain physical characteristics of the spectrum. The classification developed by Miss Maury at Harvard is, perhaps, the first attempt in this direction.

First, let us note the effect of the absolute magnitude. Three groups of stars are differentiated. 1) Very bright stars or "super"-giants. In this case c is placed in front of the capital letter B, A .... These stars are also called "c-stars". In stars later than those of A0 the lines of hydrogen and of ionized metals are unusually intense. Ca 4227 is abnormally weak when compared with Hγ or Sr$^+$ 4215. In general, all lines are very thin and sharp. The c-characteristic is not used earlier than B0. Representatives: ε Canis Majoris is a cB1, α Persei is a cF5, α Scorpii is a cM0. 2) The letter g serves for the designation of stars of large absolute magnitude (giants) of types F0-M in which the lines of oxygen and of ionized metals are too intense for a given class, and the lines corresponding to a low degree of excitation are, on the contrary, weaker. For instance, Sr$^+$ 4077, 4216 and Ti$^+$ 4290 are intense for F. Sr$^+$ 4077, 4216 are intense, and Ca 4227, 4435, and 4456 are weak for G, K, and M. Representatives: α Bootis is a/gK0 or θ Scorpii is agF0. 3) The letter d serves for the designation of stars of small absolute magnitude (the so called "dwarfs") of types F0-M. The ratio between the intensities of the lines of the ionized atoms and those of neutral atoms is approximately the inverse of that for the "giants". Representatives: α Canis Minoris is adF5, 70 Ophiuchi is adK0.

No symbols for the designation of the effect of absolute magnitude for stars of type O, B, A has yet been developed.

It should be noted that a spectral classification for "giants" may differ somewhat from a classification for "dwarf" K5 is approximately midway between K0 and M0, whereas the "giant" K5 is closer to M0.

As to the width of lines the characteristics n and s are used (n = nebulous, s = sharp).

These letters are added to O, B, A, F at the end, n corresponds to wide and washed-out lines, whereas s corresponds to sharp and narrow lines. In the c-stars the lines are always understood to be very sharp.

If in the stellar spectra are observed emission lines, i.e. bright lines, the letter e is added to the usual designations, except in those cases in which the emission is normal as, for instance, in stars of type O, P, and Q. If the emission lines in the center are noticeably reversed, the symbol er is used. If the emission lines have an absorption line on the violetside we add the symbol eq. Thus, P Cygni is designated as B4eq. The lines of the Balmer series are sometimes represented in the spectrum as bright lines, and their brightness diminishes rapidly as the distance from Hα increases. In this case the letters α, show which of these bright lines is the last. For instance for α Cygni we use the symbol A2eα.

In the case where a star is peculiar in some way the letter p is used (p = peculiar) and it characterises the preceding symbol: For instance: B2pe corresponds to a star having peculiariaties in the lines of the absorption spectrum, and B2ep corresponds to a star having peculiarities in the emission lines.

Compound spectra are usually denoted by two letters connected by the plus sign; for instance: (K0+B9).

If in the spectrum of a star a certain element is abnormal or is not identified, it is desirable to designate it separately in parentheses; for instance α Andromedae is A0p (Mn$^+$, Un). This denotes that in this star the ionized Mn is abnormal and that besides there exist unidentified lines.

In the case of "forbidden" lines, the element in question is placed within a square brackets, for instance [Fe$^+$]. If in the spectrum of stars there are noticeable emission lines not only of hydrogen but also metals, it is desirable to use the suffix "em". For instance: B0em (Cassiopeae).

It may be seen from the statements above that as our astrophysical knowledge increases the original empirical spectral classification becomes less and less satisfactory for the needs of contemporary astrospectroscopy. New symbols are constantly required to be added in order to characterize the spectrum of a star to a desirable degree. The next stage will be connected with the development of a detailed astrophysical theory of stellar spectra. This theory, operating with certain fundamental constants such as the effective temperature, the gravitational field, the coefficient of absorption, the chemical composition of stars, etc, should be able to "predict" the intensity and width of spectral lines and thus create a spectral classification. Conversely, the theory should enable us to derive these constants from the observational data as to the intensity and width of the spectral lines.

6. The System of Wave Lengths.
Catalogues of Radial Velocities.

§20. Systems of Wave Lengths. The determination of radial velocities is a very difficult problem in itself, quite aside from the complexities of the instruments employed. If we do not measure directly the shift of lines in the stellar spectrum with respect to the lines of the comparison spectrum of the same element, we obtain in measuring the spectrogram merely the wave-length of the given lines and not the radial velocities. In the case of a spectrum obtained by means of the diffraction grating the wave-lengths may be measured directly (§ 10), but in the case of a spectrum obtained by means of a prixm the wave-lengths are determined on the bases of an interpolational formula with the use of lines of the comparison spectrum of well know wave-length. The results of the measurements depend on these lines and the latter are subject to accidental and systematic errors. Let us note that for the determination of radial velocities it is necessary merely to know the relative and not the absolute wave-lengths. Doppler's formula shows that in order to determine a radial velocity we need know merely the shift $\Delta\lambda$ the value of which is based on the knowledge of the corresponding wave-length when the source of the light is at rest. The latter may be obtained in two ways both of which are not quite faultless.

One of the ways consists in the determination, with all possible accuracy, of the wave-lengths of various elements (especially those of the simplar ones such as H, He, O, Mg, etc) by means of purely laboratory investigations.

The other method is based on making use of the wave-lengths of various elements obtained as a result of measuring the solar spectrum by means of the diffraction grating. Besides, it is necessary to know the accurate values of the wave-lengths of the elements (Fe or Ti) used in obtaining the comparison spectrum. Up to very recent times Rowland's system of wave-lengths was used in astrospectroscopy. Rowland's system is based on the observations of the solar spectrum as well as on laboratory investigations of spectra made by means of the diffraction grating. However, because this system is "tied" to several washed-out sodium lines D_1 and D_2 whose wave-length were not sufficently well known at that time and also because of certain systematic errors, this system is now being replaced by the so-called international system (denoted I.A.) based on the measurement, by means of the interferometer, of the wave-length of the very sharp line of Cadmium 6438.4696 A at a pressure of 760 mm and a temperature of + 15° C. Next, the accurate values of the wave-lengths of 86 selected lines of iron, the so called secondary standards, over the entire stretch

of the visible spectrum were determined by means of the interpolational method. Finally, making use of these standard lines and using interpolation, wave lengths of 383 intermediate lines of iron - tertiary standards - were obtained by means of a concave diffraction grating. To these basic lines were gradually "tied" lines of other elements.

The I.A. system is quite satisfactory, as its accuracy is of the order of 0.002 A, an accuracy which exceeds that of the measurements in astrospectroscopy.

This system may be reduced to Rowland's system on the bases of corrections derived by Kayser. The corrections, which are often used, because both systems are still in use, are given in table 6. The accuracy of these corrections is of the order of 0.01-0.02 A.

Table 6

Portion of Spectrum	I.A. - Rowland
6850 - 6750	- 0.24 A
6750 - 6570	.23
6570 - 5500	.22
5500 - 5400	.21
5400 - 5375	.20
5375 - 5325	.19
5325 - 4550	.18
4550 - 4350	.17
4350 - 4150	.16
4150 - 3450	.15

However the difficulty of the problem for the astro-physicist is not so much in the system as in the fact that the wave-lengths given by the system are not quite applicable to stars of various spectral classes.

The spectra of stars of type B, A, and in part, of F are comparatively simple and contain the lines of H, He, O, Si, alkaline metals and earths, but only a few weak metallic lines. The spectra of these classes contain almost no "blends", i.e. close lines one on top of the other. But in a given case we should use the laboratory system of wave-length with great care, for not every shift in stellar spectra can be interpreted as a Doppler shift. The conditions of the formation

of spectra in the atmospheres of these stars are quite different from those which obtain in the laboratory. Besides, special factors may act in the stellar atmospheres (for instance, the Einstein gravitational effect causing a shift of lines toward the red end, etc.).

Had our Sun been of much greater mass with the same radius, the wave-length system, based on the measurements of the solar spectrum, would have deviated greatly from that based on the laboratory. If such a gravitational effect exists in the stellar atmospheres, the application of our laboratory system of wave-lengths may lead to an erroneous interpretation of stellar motions obtained on the bases of observed shifts of spectral lines.

This, apparently, occurs in stars of class O(and B, in part) and in the so called white dwarfs (Part III Chapter 2).

While we speak of the possible differences in the conditions of the formation of the spectrum in stellar atmospheres and in the laboratory and on their influence on the wave-length, we may mention, for instance, the Stark effect (Part II, Chapter 3), under the influence of which certain lines become asymmetrical and which leads to a change in wave-length. The Stark effect may likewise cause the appearance of "forbidden" lines (See Introduction, § 11) near the known lines of helium; with the dispersions used these lines are not quite separated and therefore the corresponding lines become asymmetric as was shown by Struve and Elvey.

Nevertheless, the laboratory system of wave-lengths may, apparently, be applied, with certain limitations, to stars of type O-B. Therefore, the laboratory values for wave-lengths are usually accepted for most lines in the spectra of these types of stars. However, for certain lines large systematic deviations are observed. For them are taken such values of wave-lengths which give the least deviations in kilometers for a mean of a series of stars (radial velocities which were obtained from all other lines are used). In the same way are treated those lines the identification of which is unknown. Finally, "forbidden" lines may appear in stars and nebulae. These lines correspond to the metastable state of the atom which cannot be reproduced in the laboratories. The wave-lengths for these lines are computed on the basis of quantum mechanics.

In stars of type A0-F5 where blends occur, especially when a spectrograph with a single prism is used the wave-lengths for numerous lines are obtained by means of reducing to their minimum value the residual differences for groups of stars of well known velocities (for certain stars these velocities may be obtained by means of a three prism spectrograph). The lines which show changes in wave-length in passing from one spectral subdivision to another are neglected. Laboratory or Rowland's values of wave-lengths may be accepted for many lines for stars of type A and, in part, for type F with a large dispersion, which is used in a three prisms spectrograph. This statement applies, in part, also to one prism spectrographs. On these principles is built the system of wave-lengths accepted by the Victoria observatory in Canada. Besides, the same observatory has developed a system of wave-lengths in international units for stars of type O and for the early subclasses of B. This system is almost entirely based on laboratory determinations.

As to the wave-lengths of the iron lines of the comparison spectrum, they are taken essentially on the bases of Burns' determinations. We also may use the secondary and tertiary standard iron lines the number of which is quite sufficient. These wave-lengths may be taken out of Trayman's book "Wave Length Tables for Spectrum Analysis".

Henceforth, by a system of wave lengths we shall mean a table of wave lengths specially selected for the determination of radial velocities.

The system of wave lengths for classes O-B and A, used at the Victoria Observatory, is of a semi-laboratory character and should be regarded as a preliminary one. With the exception of certain lines, this system gives quite accurate results for a dispersion of about 36A° = 1mm at H which is a working dispersion at present in the determination of radial velocities.

Laboratory wave lengths and Rowland's determinations (Sun) are also used in work with the three prism spectrograph and comparatively little adaptation of separate, systematically deviating lines by the method indicated above is necessary. For a very small dispersion, which is used in the determination of radial velocities of faint stars, a special system of wave length should be developed. Such a system is based on observations of stars of wellknown velocity.

Passing to stars of type G (the spectrum of which is very similar to that of the Sun) we are confronted with certain circumstances which on one hand, make the problem more complicated, yet facilitate, in part, its solution, on the other hand. Because of the appearance of a very large number of lines it is difficult even with a large dispersion to find entirely isolated lines. For the most part we have to deal here with "blends" consisting of lines of various intensity the wave lengths of which differ very little one from the other. In passing from one spectral class to another various lines will prevail in the blends which fact causes systematic shifts of the blends. Such a phenomenon was observed by Albrecht and others. In certain cases the wave length increases in passing from F to G, etc., in other cases it decreases, often having a maximum or minimum in an intermediary class. This circumstance greatly complicates the use of a single system of wave lengths for the types G - M.

Because of difficulties connected with the finding of quite isolated lines the laboratory wave lengths are not used. For the most part Rowland's wave lengths are used after they are corrected in accordance with the dispersion used, so that for the Sun and planets we may obtain velocities as close as possible to the theoretical values. It is of great advantage to be able to use for type G (i.e. for the solar type) such excellents standards of known velocity as the Sun, Mars, the Moon, and the Sky are.

Passing to the stars of type K, M we also must use solar and planetary data in spite of a certain extrapolation due to the difference in spectrum. For these stars the possibility of using laboratory wave lengths are still smaller.

Of exceptional value in the problem of determination of radial velocities for stars of type F5 - M is the application of Hartmann's spectrocomparator (§ 11). There is no doubt that the contour of a blend changes within the limits of the same spectral class, and it is often entirely impossible to compare a given blend in two adjacent classes. However, such a comparison may be realized on the spectrocomparator when certain whole sections of the spectrum containing scores of lines and blends are compared. Each blend cannot be compared directly, but if we take simultaneously a dozen of them, we may think that their individual differences compensate each other. Thus, it is possible to obtain the velocity for a series of stars of the types F5, G, K, and M by means of comparing them with the spectrum of the Sun and of the planets. This can be done with a high degree of possible accuracy, to the full extent which the computations of the Solar and planetary radial velocities

based on Celestial Mechanics permit. In this case the system of wave-lengths is of no importance, for all determinations of radial velocities are of a differential character.

This is how it is done. Velocities of maximum accuracy are determined (by means of comparison with the spectrum of the Sun and of the planets) for a series of selected standard stars of various spectral classes. Next, the spectra of the stars under consideration are compared with the spectra of the standard stars of the corresponding spectral class. This method is thoroughly reliable and has many advantages as compared with the method, described above, of the determination of radial velocities of stars of the types O-F.

Having obtained some standard stars of well known velocities, it is possible to carry out mass determinations of radial velocities of the fainter stars by the following two methods. The first one consists in the usual comparison of the spectra with a standard star on the spectrocomparator (§ 11). However, sometimes it is desirable (because of technical difficulties or because a spectrocomparator is not readily available) to use the ordinary measuring devices. These technical difficulties may result, for instance, from systematic errors that will occur if the spectrogram of the given star differs greatly in exposure from that of the standard star. Next, the spectrocomparator demands higher quality of spectrogram with respect to the width of the spectrum and exposure as compared with that for ordinary measurements. Therefore, certain observatories, Mount Wilson for instance, prefer to determine the radial velocities of G-M types of stars by means of the ordinary microscope-micrometer.

The second method is as follows: having measured the spectrograms of standard stars of well known velocities, the wave-lengths for whole series of lines are determined, and only those of them are left which do not show any changes in passing from one spectral class to another. Next, this system may be somewhat improved on the bases of numerous measurements by using the deviations in radial velocities for individual lines and reducing the systematic deviations to zero. This method is, in part, similar to the method of adaptations of individual wave-lengths in stars of types O-F.

The method of standard velocities also has the advantage that the unavoidable systematic errors, due to the instruments, in radial velocities are reduced to a minimum, as the radial velocities of the standard stars themselves are determined by a purely differential method with respect to the Sun and the planets.

Curtiss' method is a further development of the method of standard velocities. For a series of standard stars of well-known velocities or for the Sun readings for separate lines (and not for the wave lengths) are taken. The readings of the lines of any star are compared with those of the table of normal readings, and then they are reduced to the same dispersion by means of the comparison spectrum. Next the velocities of the star under investigation is obtained in the usual way by comparing it with a standard star. At present very small dispersions, often of the order of 90A = 1mm near $H\gamma$ are used. For such a small dispersion the selection of the wave-length is of little importance. Besides, the systematic errors due to the instruments are very large here. Therefore, the method of standard velocities is the only practical method for such small dispersions.

In the case of large dispersions (in a three prism spectrograph and in part, in a one prism spectrograph) we may proceed also by a comparison with Rowland's atlas, selecting the isolated lines for which the Rowland values are known. This method is sufficiently good for the solar type of stars, but it becomes doubtful for stars of types K-M unless the system is corrected on the basis of the study of the residual differences for the individual lines by numerous measurements.

It is evident that the derived system of wave lengths for a given instrument is definitely connected with the systematic errors belonging to the given instrument. If a single system of wave lengths were used for all instruments, such a system, however desirable in principle, would have caused systematic errors for each of the instruments. On the contrary, it is possible to reduce the systematic error to a minimum by developing a special system for each instrument. Contemporary astrospectroscopy has not yet definitely solved this problem. In reality, almost every major observatory uses to a certain degree its own system of wave lengths; nevertheless, the radial velocities suffer more or less from systematic errors. Usually, in the catalogues of radial velocities, which were obtained at different observatories, there are numerous stars in common; the systematic errors are derived by comparing these common stars. The systems of the Lick and Victoria Observatories may be regarded at present as the best (§21).

System I. A. is based on the wave length of the red line of cadmium determined at 760 mm and 15° C. When the temperature and pressure change the wave length also changes, for it depends on the medium through which the vibrations pass. Sometimes it is necessary to reduce the wave-lengths to standard conditions (the reduction of wave length to a vacuum). For this purpose it is necessary to study the refraction of the air for various wave lengths and the dependence on density, pressure and temperature. The investigations of Kayser and Runge, and especially the latest measurements of Meggers and Peters, enable us to construct tables for the reduction to a vacuum. Such a table may be found in Twyman's book, which we mentioned previously. For instance, for λ3600 and 6600 (I.A.) these corrections are respectively equal to 1.0233 and 1.8193 A. Introducing these corrections we obtain wave lengths which are inversly proportional to the frequencies of the vibrations. A transfer to the frequencies of the vibrations is necessary in the problem of spectral series.

Since in stellar astronomy the determination of wave lengths is often connected with the comparison spectrum, the system will be the same as for the latter. For the same reason the determination of wave lengths does not depend on the pressure and temperature of the air, although these corrections are practically very slight.

§21. Catalogues of the Radial Velocities of Stars. The first determinations of radial velocities were made by the visual method by Keeler (at the Lick Observatory in 1890), who accurately obtained the velocities for three stars. The earlier attempts of certain English and German astronomers led to very doubtful results. The application of the photographic method opened a new era in the problem of the determination of radial velocities of stars. The pioneers were Vogel and Scheiner at Potsdam (1889-1891) and A. Belopolsky at Pulkovo (1890). The computed error of the radial velocity appeared to be ± 2.6 km., but, because of a systematic error, the accuracy of the results of the Potsdam observers was much smaller. A greater increase in accuracy was achieved by W. Campbell at the Lick Observatory (1895-96). This increase was due to a specially constructed spectrograph and to the excellent 36-inch refractor.

The error for the bright stars was equal to only ± 0.5 km, and for bright stars with good lines the error was only ± 0.5 km. This accuracy remains unexcelled up to the present time. Further successes consisted merely of reaching such a degree of accuracy for the fainter stars; these improvements were due chiefly to a better construction of spectrographs in regard to flexure (§8) and temperature (§13), to a greater transparency of the prisms, and to the increase in power of the instruments.

We may also note the decrease in dispersion from earlier days to the present time, but the accuracy decreased in a much smaller proportion. In certain spectral classes (stars with blurred and faint lines) the transfer from the three prism spectrograph to a one prism spectrograph has not lessened the accuracy at all, although the dispersion decreased to 1/3 of its former value. Next, it is often of more practical value to obtain several spectrograms in a given interval of time than to obtain only one spectrogram needing a longer exposure. Finally, the reason for coming down to small dispersions is the desire to work on the fainter stars and fainter objects of special interest.

The results of observations of radial velocities are given in various catalogues the comparison of which enables us to investigate certain systematic errors.

We emphasized in the previous sections the complexity of the theoretical and technical parts in the determination of radial velocities.

It is almost impossible to draw a line between accidental and systematic errors. Small systematic errors which cannot be clearly differentiated will enter into the catalogue as accidental errors. However, the summary effect of the systematic errors may be made apparent in two ways:

1) For stars of the solar type by means of comparing the theoretical and observed velocities of the Sun, planets, Moon, and the sky; 2) by comparing the catalogues of radial velocities of various observatories, containing the same stars.

We enumerate below only the chief sources of systematic and accidental errors:

1) the character and number of spectral lines in the spectrum;
2) the unsatisfactory centering of the telescope and spectrograph;
3) differential bending of the spectrograph;
4) general bending in the spectrograph;
5) general bending in the telescope and the change of collimation;
6) atmospheric dispersion;
7) temperature effect;
8) the width of the slit and the asymmetric "guiding" of the star on the slit;
9) deficiencies in the optics of the telescope, especially the existence of astigmatism;
10) the heterogenuousness of the prism and deficiencies of its surface;
11) the "coma" effect of the objective of the camera;
12) the errors of the focus of the camera in connection with the uneven illumination of the objective of the collimater;
13) the error of the focus of the camera in connection with a larger inclination of the camera holder;
14) the system of wave lengths;
15) the variable velocity of stars;
16) the personal equation;
17) the temporary systematic errors of unknown origin.

Most of the errors indicated here have been considered by us previously in the respective sections.

The characteristic peculiarities of each catalogue of radial velocities are its accidental and systematic errors. Besides those causes indicated here, the dispersion and the spectral class of the stars are of importance, which determine, in part, the number and character of the lines. The inner accuracy is chiefly determined by accidental errors (in catalogue they are called the probable or mean errors of velocity) which include also all the systematic errors not previously taken into consideration. When there is a small number of spectrograms, as is usually the case, the probable error is best computed by Peters' formula: $\pm\frac{0,845\Sigma\Delta}{n\sqrt{m-1}}$ and not by the formula of squares.

Below we give, as an illustration, the probable error of one spectrogram and the mean velocity of the star. It is given by a very good catalogue based on observations made at the Victoria Observatory by a one-prism spectrograph:

	Stars of types G-M and, in part, of type F, having good lines		Stars of types B-A having good lines, and those of type F having washed out lines		Stars of type A and O-B having washed out lines	
	From	To	From	To	From	To
ts of probable errors single spectrogram	± 0.2km	± 2.5km	± 1.2km	±3.5km	±2.5km	± 10.0
ts of the probable r of the mean velocity	± 0.1km	± 1.0km	± 0.5km	±1.5km	±1.0km	± 3.8

For three prism spectrograph the error for the third group remains the same as in the table, and for the first and, in part, for the second groups the error decreases almost by half.

The real error (accidental and systematic) is characterised not so much by inner agreement as by a comparison with the results obtained at other observatories. Although no general system of wave-lengths has as yet developed, and although it is not yet known which observatory is the most accurate with respect to radial velocities, nevertheless we may, according to all indications, take the results of the Lick Observatory as a standard of comparison. The Lick system of radial velocities has remained homogeneous for almost thirty years, and besides it is connected to the dispersion of a three prism spectrograph (about 10A = 1mm at H_γ).

The tables below show the results of the comparisons for three groups of stars: A5-M, O-B5, and B8-A3.

Stars of Type A5 - M

Observatories	Number of Stars	Mean deviation in km.
Cape - Lick	349	+ 0.20
Mount Wilson - Lick	254	+ 0.34
Victoria - Lick	130	- 0.71
Yerkes - Lick	17	+ 0.70
Bonn - Lick	215	+ 1.49
Pulkovo - Lick	12	- 0.14
Simeis - Lick	17	+ 0.50 (G-M)
Detroit - Lick	16	- 1.97
Potsdam - Lick	9	- 1.49
Lowell - Lick	10	+ 0.72
Ottawa - Lick	7	- 1.01
Allegheny - Lick	4	- 3.80

In the following tables the asterisk* shows that stars having deviations of more than 11 km were eliminated (tables follow on the next page)

Of late, the commission on radial velocities of the International Astronomical Union (I.A.U.) spectra are recommended to be photographed at all observatories which are engaged in the determination of stellar radial velocities.

Below is given a list of the more important catalogues containing altogether more than 3000 stars.

Lick. Radial velocities of about 2600 stars of the northern and southern skies brighter than 5.55 (fainter stars appear rarely). Public. of the Lick Observatory. Vol. XVI-1928.

Stars of Type O5 - B5

Observatories	Number of Stars	Mean deviation in km.	Mean deviation regardless of sign in km.
Yerkes - Lick	149	+ 0.88	8.7
Yerkes - Lick*	107	+ 0.22	4.0
Victoria - Lick	8	- 6.65	8.0
Victoria - Lick*	6	- 2.65	4.4
Mount Wilson - Lick	33	- 0.56	4.2
Mount Wilson - Lick*	30	- 0.14	3.3
Ottawa - Lick	22	- 5.97	6.8
Ottawa - Lick*	20	- 5.18	6.1
Detroit - Lick	8	+ 2.54	5.4
Detroit - Lick*	7	- 0.65	2.5

Stars of Type B8 - A3

Observatories	Number of Stars	Mean deviation in km.	Mean deviation regardless of sign in km.
Yerkes - Lick	81	+ 3.41	7.4
Yerkes - Lick*	64	+ 1.63	4.0
Victoria - Lick	58	- 3.98	6.4
Victoria - Lick*	49	- 2.48	4.0
Mount Wilson - Lick	86	+ 0.72	5.3
Mount Wilson - Lick*	75	+ 0.25	3.3
Ottawa - Lick	17	- 2.1	5.3
Ottawa - Lick*	14	- 2.2	3.3
Detroit - Lick	11	- 1.75	4.3
Detroit - Lick*	11	- 1.75	4.3
Allegheny - Lick	16	- 3.62	6.7
Allegheny - Lick*	14	- 4.24	5.8

Mount Wilson. Radial velocities of 1013 stars, chiefly of types F, G, K, M. With respect to the brightness of the stars the catalogue is not homogeneous (from the brightest stars to those of the 8-th magnitude and fainter). Astrophysical Journal 57, 1923. Radial velocities of 741 stars. The character of the catalogue is the same as that above. Astrophysical Journal 70, 1929.

Victoria. Radial velocities of 594 stars. Stars from $4^{m}.6$ to $6^{m}.9$ (rarely of $7^{m}.0$) of various spectral classes. Public. of the Dominion Astroph. Observatory Vol. II, 1. 1921.

Victoria. Radial velocities of 523 stars. Stars of type O-B5 up to $7^{m}.25$. Public. of the Dominion Astroph. Observatory. Vol. V, I.

Yerkes. Radial velocities of 368 helium stars (mostly brighter than $5^{m}.25$). Astrophysical Journal 64, 1926.

Yerkes. Radial velocities of 500 stars of type A (mostly brighter than $5^{m}.25$) Publ. of the Yerkes Observatory. Vol. VII, Part I.

Simeis. Radial velocities of 301 stars from $5^{m}.5$ to $6^{m}.7$ (mostly stars of type A-F). Works of Pulkovo Observatory, vol. 43, 1933.

In Voute's catalogues: First Catalogue of Radial Velocities (2071 objects) and Second Catalogue of Radial Velocities (4032 objects) we may find a list of radial velocities of stars, nebulae, and clusters determined up to 1928. The velocities of these catalogues are not reduced to any system.

The Lick Observatory will soon publish a collated catalogue of radial velocities of about 6700 objects. Public. of the Lick Observatory. Vol. XVIII.

7. The Investigation of the Planetary System.

§22. The Spectrographic Method of Investigation of Planets. These are two peculiarities of method in the application of the spectrograph to the study of planets: 1) the slit must be sufficiently long to obtain the spectrum of any section of a planet; 2) we must be able to rotate the entire spectrograph with the slit so that the slit may be set parallel to any section of the disk of the planet.

The conditions for obtaining a spectrum of the planets and of elongated objects are somewhat different from those for the stars. As we have to deal here with surface brightness, the dimensions of the instrument are of secondary importance. Next, the quality of the image has almost no influence on the time of the exposure of a planetary spectrum. As the planets show a continuous spectrum with absorption lines (the reflected solar spectrum), the condition, characterizing the brightness of the spectrum depending on the constants of the spectrograph, remain approximately the same as those for stars (§3). It is desirable that the slit should have a certain curvature so designed that the spectral lines for the spectrum will be straight. This is especially important in spectrographic investigations of planetary rotation. As the changes in the solar spectrum which is reflected by the planets and which is determined by the passing of the light through the planetary atmosphere, are almost exclusively in the visual and the infra-red part, it is desirable to use, in the investigations of planetary atmospheres, a spectrograph which is suitable for the study of these regions.

It is important in spectrophotometric investigations that the chromatic curve should deviate but little from a plane along the entire spectrum, so that the focus may be practically the same for the entire spectrum.

One of the chief applications of the spectrograph to planets is the study of the rotation of planets. We shall state here a few formulae for the study of the effect of the rotation of planets. They will show us the direction in which the method is developing.

According to Doppler's principle in its application to reflected light, the shift of the spectral lines is determined by the total effect of the motion of the reflecting surface with respect to the Sun (as the source of light) as well as with respect to the observer. Instead of

$$\lambda_1 - \lambda = \frac{v}{c}\lambda$$

we have:

$$\lambda_1 - \lambda = \left(\frac{dR_1}{dt} + \frac{dR_2}{dt}\right)\frac{\lambda}{c}, \qquad (50)$$

where R_1 and R_2 are the distances of the planet from the Sun and the Earth, and $\frac{dR_1}{dt} = v_1$ and $\frac{dR_2}{dt} = v_2$ are radial velocities with respect to the Sun and Earth.

Let us introduce the following designations: 1) α is the angle at the center of the planet between the directions toward the Sun and toward the Earth; 2) ψ_1 and ψ_2 are the angles between the equatorial plane of the planet and the directions toward the Sun and toward the Earth; v_0 is the equatorial velocity of the rotation of the planet in km/sec; v_a and v_c are the spectrographic velocities of the points A and C of the two edges of the disc at the equator. Evidently,

$$\left.\begin{aligned} v_a &= v_0\cos\psi_1 + v_0\cos\alpha\cos\psi_2 \\ v_c &= -v_0\cos\alpha\cos\psi_1 - v_0\cos\psi_{1,2} \end{aligned}\right\} \qquad (51)$$

In these formulae we neglect the same terms (for both edges) which depend on the motion of the planet as a whole with respect to the Sun and to the Earth.

Then the observed differences of velocities will be equal to:

$$v_a - v_c = v_0(1+\cos\alpha)(\cos\psi_1 + \cos\psi_{1,2}) \qquad (52)$$

Near opposition is very close to therefore,

$$v_a - v_c = 2v_0(1+\cos\alpha)\cos\psi_1 \qquad (53)$$

At the epoch of opposition or conjunction $\alpha = 0$, and hence

$$v_a - v_c = 4v_0\cos\psi_1. \qquad (54)$$

These formulae apply only when the slit of the spectrograph intersects the disk of the planet along the equator of the planet.

When it does not, it is then necessary to know the angle η between the circles of declination and those of latitude. This angle determines the orientation of the slits with respect to the ecliptic.

$$\operatorname{tg}\eta = \operatorname{tg}\varepsilon\cos\odot \qquad (55)$$

(ε is the inclination of the equator to the ecliptic, $\odot$ is the longitude of the Sun). It is assumed here that the slit coincides with the diurnal circle.

Next, it is necessary to know the angle μ between the latitude circles and the circle perpendicular to the plane of the equator of the planet.

$$\mathrm{tg}\,\mu = \mathrm{tg}\, i \cos(P - \Omega). \qquad (56)$$

where i is the inclination of the equator of the planet, is the longitude of the ascending node of the equator of the planet, and P is the longitude of the planet. Having computed the angle μ -η, we may place the slit accurately with respect to the equator of the planet. The method of determination of the rotation of the planet according to the observed difference of the spectrographic velocities of the two sides of the planet is not faultless, as the limbs of the planet are somewhat washed out because of the atmosphere, and therefore the edges of the spectral lines corresponding to the limbs are not determined accurately. Next, in the case of astigmatism in the spectrograph, errors of instrumental nature ensue.

The angle φ which determines the inclination of the spectral lines in the spectrum of the planet with respect to lines of the comparison spectrum (Deslandres' method), can be determined more accurately. Let x and y be the coordinates of a certain point on a line in the spectrum of a planet and let v be the velocity (along the line of sight) of the corresponding point. Next, let D be the linear dispersion of the spectrograph near the given line, and let 2p be the width of the spectrum of the planet in milimeters. Evidently

$$y = x\,\mathrm{tg}\,\varphi. \qquad (57)$$

On the other hand, y is the linear shift corresponding to Doppler's effect, and hence for the planet it will be:

$$y = \frac{2v\lambda\cos\psi}{c - D}. \qquad (58)$$

where is the angle between the plane of the equator and the line Planet-Earth or Planet-Sun (near opposition this angle may be considered as equal for the Sun and the Earth) and c is the velocity of light. For a point on the limb of the disk x = ρ and we shall obtain the velocity v_o of the point of the equator along the line of sight from the following formula:

$$v_0 = \frac{CPD\,\mathrm{tg}\,\varphi}{2\lambda\cos\psi} \qquad (59)$$

Measuring the inclination φ we may thus determine the velocity of the rotation of the planet for any point on the limb of the disk. If the measuring apparatus has a small reading circle, it is possible by rotating the eye-piece end of the microscope, to make the cross-hair coincide with the lines of the planetary spectrum and then also with the lines of the comparison spectrum; the difference will determine the angle φ.

The quantity ρ (the half-width of the planetary spectrum) is equal to $d\frac{f}{f_1}$, where d is the computed semidiameter of the planet and f and f_1 are the focal lengths of the camera and collimator (the measured ρ cannot be very accurate).

This method is also used for the determination of the rotation of Saturn's rings.

We said in § 15 that a comparison of observed and theoretical velocities of the solar bodies is an excellent way to study the systematic errors of the determination of radial velocities. At the same time the spectra of the Sun, the planets, the moon, and the sky are excellent standards in the determination of radial velocities of stars of the type G-M. We shall show now how these velocities are computed for the planets.

When the position of the slit on the disk of a planet is properly selected, there is no need to use the correction for the rotation. The velocity of the planet is computed by formula (50), R_1 and R_2 are given in "The American Ephemeris" and the "Nautical Almanac".

Everything is reduced to the differentiation of the interpolation formula (60) for R.

Let us have the following little table

$T-2\omega$	$f(T-2\omega)$				
		a''			
$T-\omega$	$f(T-\omega)$		b'		
		a'		c'	
T	$f(T)$		b	c	d
$T+\omega$	$f(T+\omega)$	a_1		c	
			b_1		
$T+2\omega$	$f(T+2\omega)$	a_1''			

Here $a = \frac{a'+a_1}{2}$, $c = \frac{c'+c_1}{2}$

Let the interpolation formula have the following form:

$$\log R = f(T+t) = f(T) + \left(a - \frac{c}{6}\right)n + \left(\frac{b}{2} - \frac{d}{24}\right)n^2 + \frac{c}{6}n^3 + \frac{d}{24}n^4 + \dots, \qquad (60)$$

where $n = \frac{t}{w}$

The change is $\frac{\delta \log}{\delta t} = \frac{\delta}{\delta t} f(T+t)$

or

$$\frac{\log e}{R}\frac{\delta R}{dt} = \frac{1}{\omega}\left(a - \frac{c}{6}\right) + \left(b - \frac{d}{12}\right)n + \frac{c}{2}n^2 + \frac{d}{6}$$

where log e = 0.4343.

The velocity at the moment T + t is expressed in astronomical units of distance and in units of time ω, i.e. in mean solar days.

In the transformation of the velocities into km/sec we have

$$\frac{\delta R}{\delta t} = v \, KM/sec = \frac{149500000}{\omega \log e} R \left[\left(a - \frac{c}{6}\right) + \left(b - \frac{d}{12}\right) n + \frac{c}{2} n^2 + \frac{d}{6} n^3 \cdots \right] \quad (62)$$

In the logarithmic form it will appear as follows:

$$\log v \log \frac{14950000}{\omega \log c} + \lg R + \log [\cdots\cdots] . \quad (63)$$

ω is expressed here in seconds of time; log R is taken according to formula (60). Fo r the first term we have:

ω = 1 Solar days ... $\log \frac{149500000}{\omega \log e}$ = 3.6003

ω = 2 " = 3.2993

ω = 4 " = 2.9983

ω = 8 " = 2.6973

§23. The Determination of the Solar Parallax by the Stellar Radial Velocities

The spectrographic observations give the radial velocity of a star with respect to the observer. In order to compare the radial velocities (of a given star) obtained at different epochs, it is necessary to reduce them to the Sun. As was indicated in §11, we take into consideration here a correction due to the revolution and rotation of the Earth. Besides, we also must take into consideration the perturbations of the Earth by the planets and the Moon.

We may pose a reversed problem: to determine the correction for the Earth's revolution and rotation and thus to determine the size of the Earth's orbit and the solar parallax from the observations of the radial velocities of a star which is, let us say, in quadrature with the Sun. Such a method of determination of the Solar parallax was proposed in the early period when radial velocities were determined for the first time.

The corrections for the rotation of the earth and its perturbation by the Moon and the major planets are very small and can be computed with an accuracy of one thousandth of a kilometer. For the correction of the rotation of the Earth we have the following formula

$$v_a = -0^{km}.465 \, t \cos\delta \cos\varphi .$$

It is easy to obtain formulae for the corrections of v_m and v_p for the lunar and planetary perturbations:

$$v_m = -0^{km}.14 \sin(\lambda - ☾) \cos\beta$$
$$= -0^{km}.012 \sin(\lambda - L_1) \cos\beta$$
$$= -0^{km}.003 \sin(\lambda - L_1) \cos\beta$$

Here λ and B are the longitude and latitude of the star, ☾ is the longitude of the moon, and L is the longitude of the planet.

Having a series of observed velocities v_o obtained at various epochs for a given star, we may obtain the velocity of the star v_s and the corrections for the orbital motion of the Earth v_a. The latter enables us to determine the velocity of the Earth V_a from the following formula

$$v_a = -V_a \sin(\lambda - ⊙ + i) \cos\beta.$$

Here

$$tg\, i = \frac{e \sin(⊙ - \pi)}{1 + e\cos(⊙ - \pi)};\quad V_a = \frac{a}{\sqrt{1-e^2}}\,\frac{2\pi}{T}\left[1 + e\cos(⊙ - \pi\right.$$

where e is the eccentricity of the terrestrial orbit, a is the major semiaxis, T is the number of solar seconds in a sideral year, π is the longitude of the Sun at the perihelion.

Having determined a, we shall find the solar parallax p:

$$a = \frac{A}{p \sin 1''},$$

where A is the semiaxis of the Earth.

The best determination of the parallax of the Sun on the bases of numerous observations of radial velocities for seven stars was made at the Cape (Cape of Good Hope) Observatory, and its value is:

8".802±0."004

The mean value of the solar parallax obtained by various methods is 8".803± 0".004. The mean constant of aberration (knowing the velocity of light and of the Earth) has been also determined by various methods, and the value: 20".473±0.015 is very close to the accepted value 20".47. Thus, it is possible to determine accurately by means of radial velocities one of the most fundamental of astronomical constants.

Ka

Fr

Bel

Can

Har
("T

Wri
ped

Plas

Hart

Hart
Ster

Camp

Flas

Campl

Flasl
ntro

Schwa
bje

Wadsw
ppli

Perri
pect

Wright
scen

BIBLIOGRAPHY.

Kayser, Handbuch der Spectroscopie, V. I.

Frost - Scheiner, Astronomical Spectroscopy.

Belopolsky, A. A. Astrospectroscopy (In Russian).

Campbell, The Mills Spectrograph ("The Astrophysical Journal". Vol. 8, 1898.)

Hartmann, Remarks on the Construction and Adjustment of Spectrographs ("The Astronomical Journal" V. 11, 1900, V. 12, 1900.)

Wright, Description of the Instruments and Methods of the D.O. Mills expedition ("Publications of the Lick Observatory", V. 9, 1905).

Plaskett, Report of the Chief Astronomer (Ottawa, 1906, 1907, 1908, 1909, 1910).

Hartmann, The Spectro-comparator ("The Astronomical Journal", V. 24, 1906.)

Hartmann, Über die Ausmessung und Reduktion der photographischen Aufnahmen von Sternspektren ("Astronomische Nachrichten". V. 155, 1901).

Campbell, "Publications of the Lick Observatory" V. 16, 1928. Introduction.

Plaskett, "The Astronomical Journal" V. 44, 1919.

Campbell, Stellar Motions.

Plaskett. Publications of the Dominion Astrophysical Observatory V. 5 #1. Introduction.

Schwarzschild, Beitrag zur Bestimmung von Radialgeschwindigkeiten mit dem Objektivprisma ("Public. Astrophys. Obs. zu Potsdam". #69.)

Wadsworth, Description of a New Type of Focal Plane Spectroscope and its Application to Astronomical Spectroscopy ("The Astrophysical Journal" V. 15, 1902).

Merrill, A Plane grating Spectrograph for the Red and Infra-red Region of Stellar Spectra ("The Astrophysical Journal" V. 74, 1931).

Wright, The Quartz Spectrograph of the Lick Observatory and some Observations recently secured with it (Lick Obs. Bulletin. #291).

YE 17842

14 DAY USE
RETURN TO DESK FROM WHICH BORROWED
ASTRONOMY LIBRARY
This book is due on the last date stamped below, or
on the date to which renewed.
Renewed books are subject to immediate recall.

~~MAR 19 1965~~
DEC 11 1967
DEC 1

LD 21-100m-6,'56
(B9311s10)476

General Library
University of California
Berkeley